Transport of Dirac Fermions in Presence of Spin-orbit Impurities

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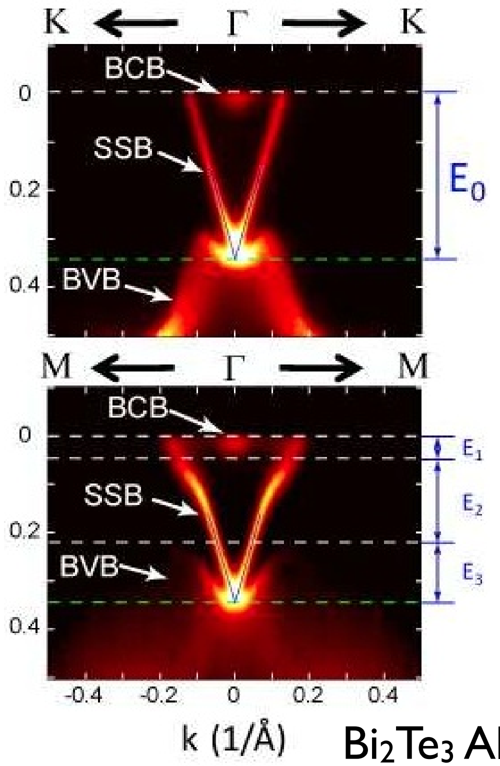
DPG Frühjahrstagung 2014, Dresden

Outline

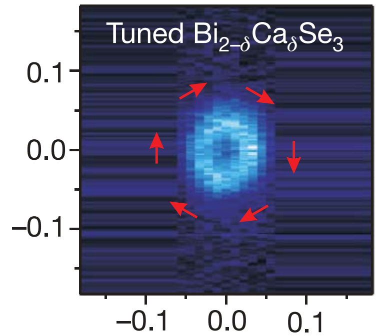
- Introduction to transport in 3D topological insulators
    - 3DTI surface states and transport
    - Regime of coherent transport (weak localization)
- Effects of spin-orbit impurities in 3DTI
    - Elastic scattering time
    - Diffusion constant
    - Quantum correction to conductivity
- Perspectives

3D Topological insulators surface states

- 3DTI : insulator with odd number of topologically protected surface states (Bi_2Te_3 , Bi_2Se_3, strained HgTe...)



Bi_2Te_3 Alpichshev *et al.*, 2010

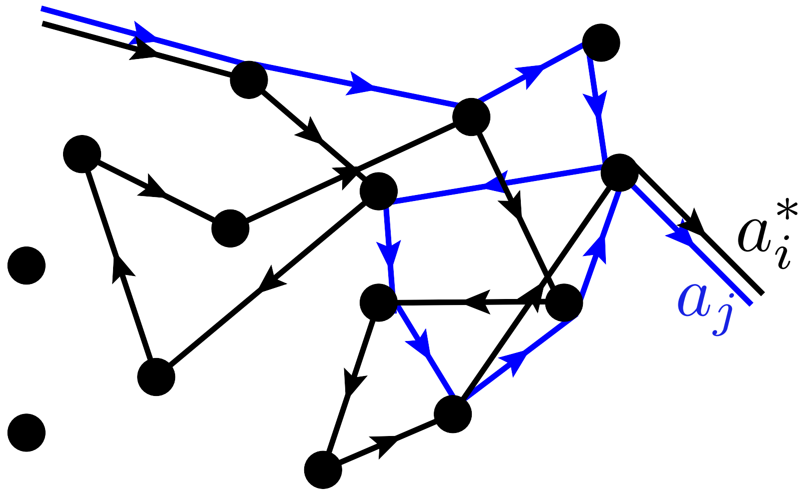


D. Hsieh *et al.*, 2009

- Strong spin-orbit coupling : Spin-momentum locking
- Dirac fermions Hamiltonian : $\mathcal{H} = \hbar v_F (\vec{k} \times \vec{\sigma})_z$

Transport in mesoscopic physics

- Mesoscopic physics = weak disorder, coherent transport $\lambda_F \ll l_e \ll L, L_\phi$
- Scattering of the electrons on impurities
- Each trajectory has a given probability amplitude a_i
- Conductivity $\sigma \propto \sum_{i,j} a_i^* a_j$



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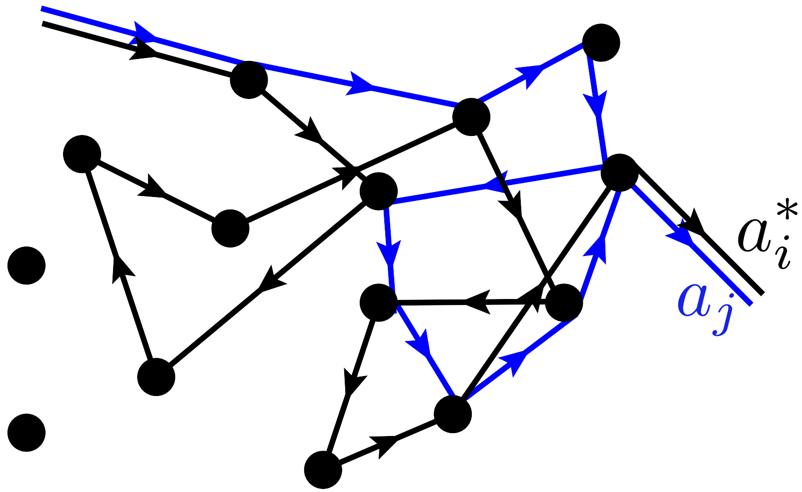
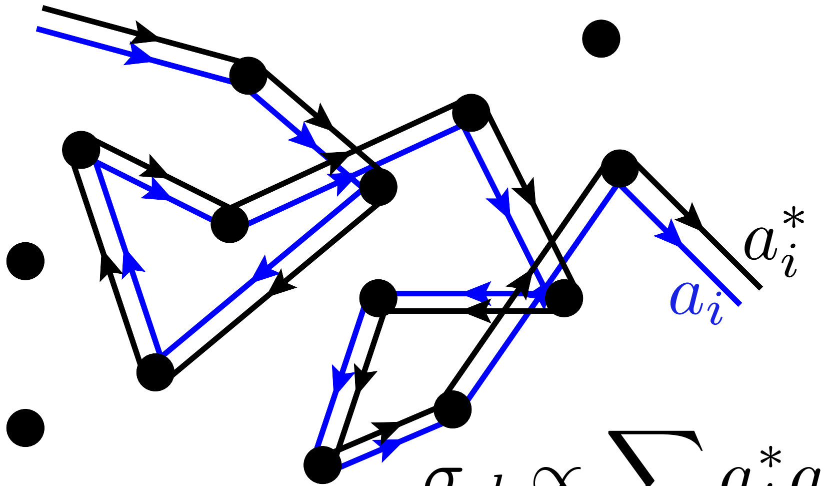


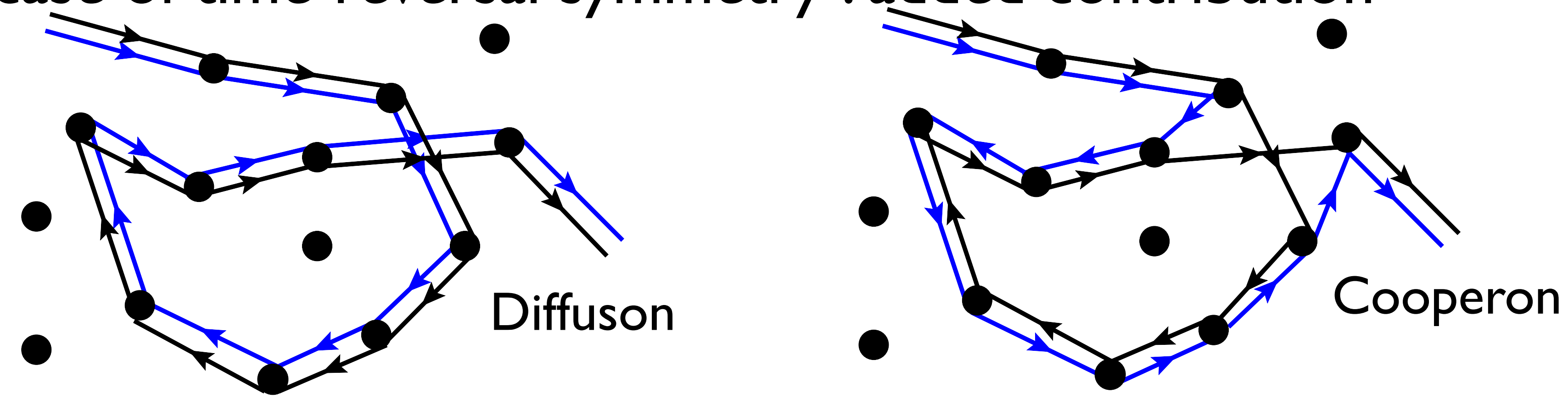
$$\longrightarrow 0 \quad (k_F \ell_e \gg 1)$$



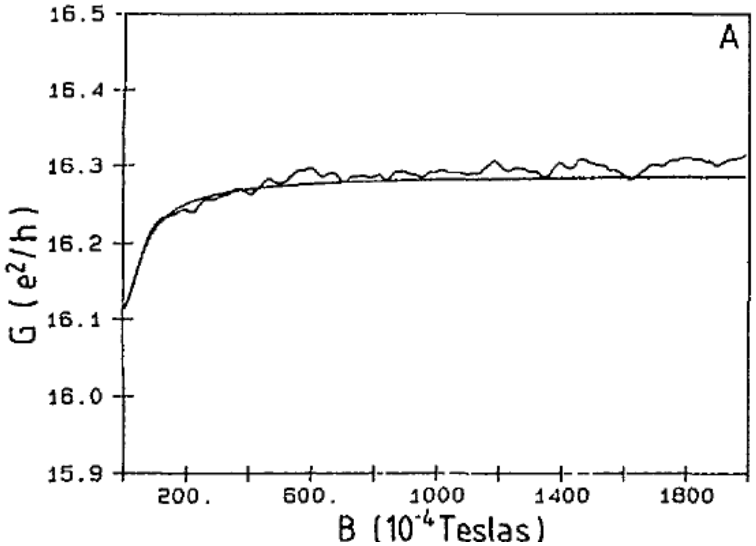
$$\sigma_{cl} \propto \sum_i a_i^* a_i$$

Quantum corrections to conductivity

- In case of time-reversal symmetry : added contribution



- Constructive interference suppressed by magnetic field : Weak localization (Altshuler *et al.*, 1980)



Weak anti-localization

- In presence of spin orbit impurities (Hikami *et al.*, 1980)

$$V(\vec{k},\vec{k}') = U(Id + i\lambda(\vec{k}\times\vec{k}').\vec{\sigma})$$

- Elastic scattering time modification

$$\frac{1}{\tau_e} = 2\pi\rho(E_F)n_I U^2(1+\lambda^2 k_F^4)$$

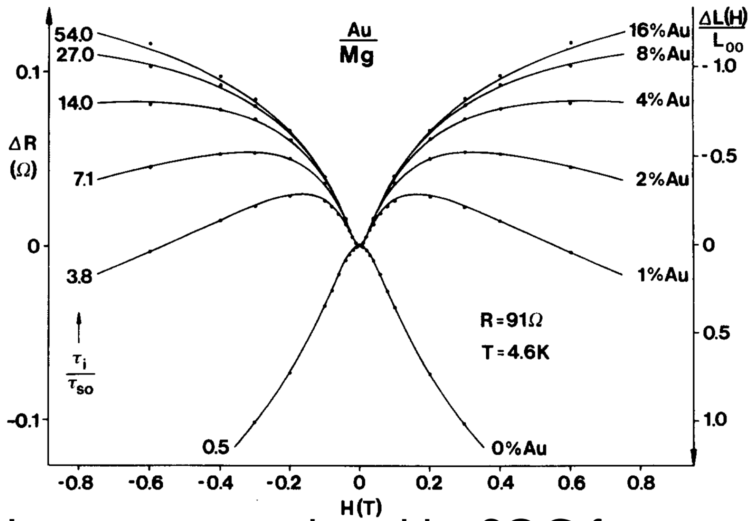
- Quantum correction to conductivity : $\sigma = \sigma_{cl} - \frac{\alpha e^2}{\pi^2\hbar}\ln L$

2 spin 1/2 : 4 cooperon modes

- 3 triplet (+1/2, killed)

- 1 singlet (-1/2, preserved)

$$\alpha \ : \ 1 \rightarrow -1/2$$



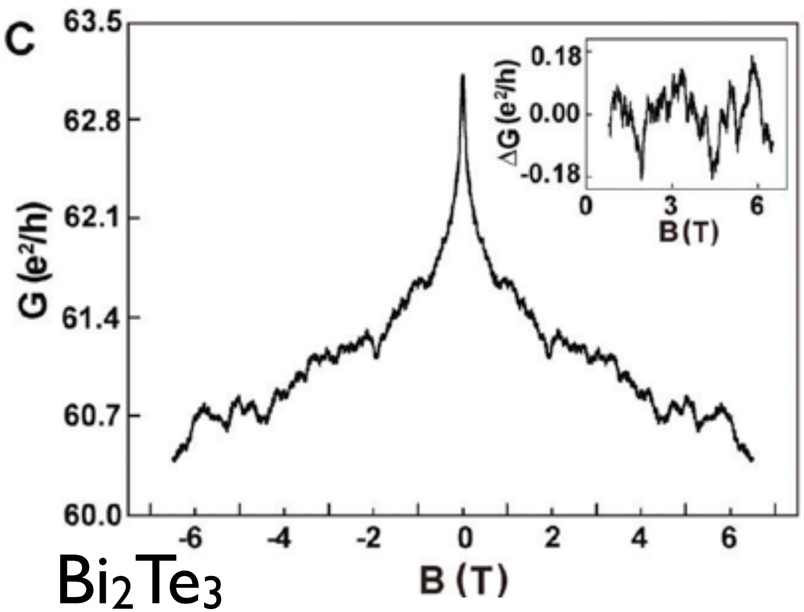
WL to WAL correction induced by SOC from impurities for a 2DEG with parabolic dispersion

Weak anti-localization!

Coherent transport of Dirac fermions

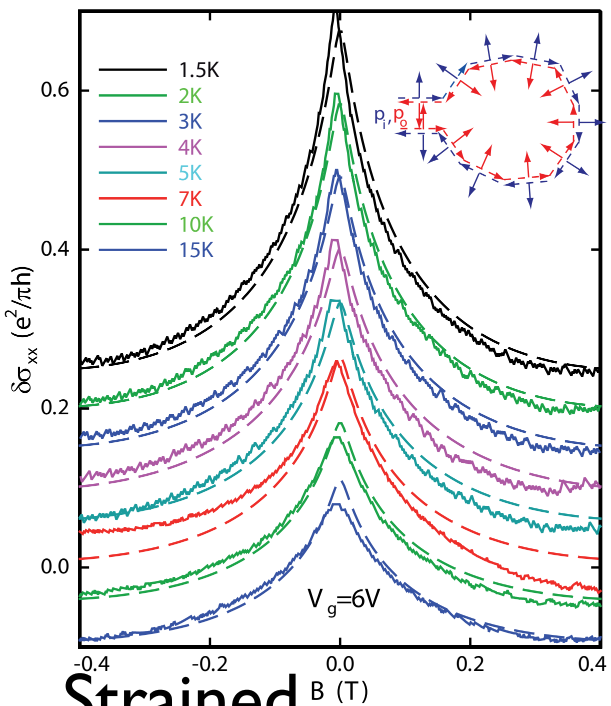
- Dirac fermions + scalar disorder : weak anti-localization

th : Tkachov and Hankiewicz, PRB 84 (2011)
Adroguer et al., NJP 14 (2012)



Bi_2Te_3 thin film

Kong et al., 2010



Strained HgTe

Bouvier et al., 2011

- What is the effect of the spin-orbit impurities on the Dirac fermions physics?

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Elastic scattering time

- Model : Dirac fermions + weak scalar disorder + SOC from impurities

$$\mathcal{H} = \hbar v_F (\vec{k} \times \vec{\sigma})_z + V(\vec{k}, \vec{k}')$$

$$V(\vec{k}, \vec{k}') = U(Id + i\lambda(\vec{k} \times \vec{k}').\vec{\sigma})$$

- Elastic scattering time via Fermi golden rule

$$\frac{1}{\tau_e} = \pi\rho(E_F) n_I U^2 \left(1 + \lambda k_F^2 + \frac{\lambda^2 k_F^4}{2}\right)$$

- Self energy calculation

$$\frac{1}{\tau_e} = \pi\rho(E_F) n_I U^2 \left(1 + \lambda k_F^2 + \frac{\lambda^2 k_F^4}{2}\right)$$

- New : Linear dependance in λ of the elastic scattering time!

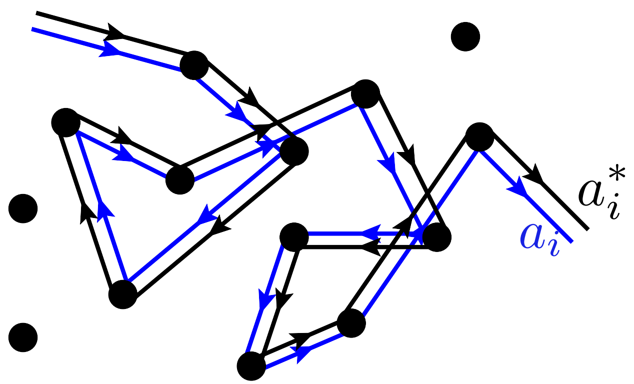
Diffusion constant $\sigma = e^2 \rho(E_F) D$

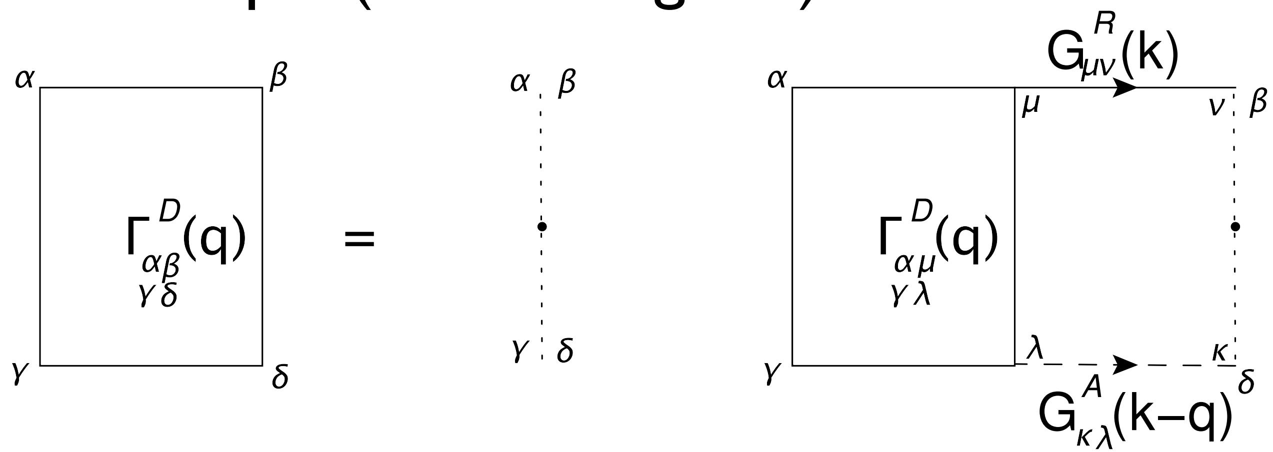
- Solving the kinetic equation

$$-e\vec{E}.\vec{\nabla}_{\vec{k}} f = \int \frac{d\vec{k'}}{(2\pi)^2} 2\pi |\langle \vec{k'}|V|\vec{k}\rangle|^2 \delta(E(\vec{k'}) - E(\vec{k}))(f(\vec{k'}) - f(\vec{k}))$$

$$\sigma_{cl} = e^2 \rho(E_F) v_f^2 \tau_e (1 - \lambda k_F^2 + o(\lambda))$$

- Standard diagrammatic technique (ladder diagram)



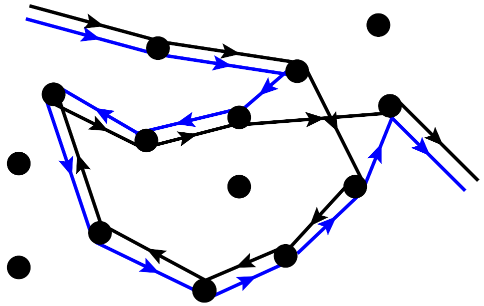


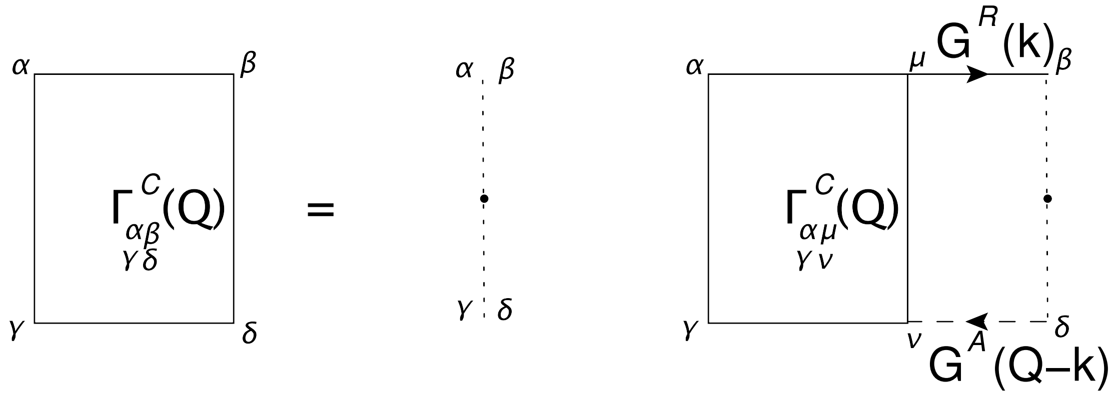
$$\sigma_{cl} = e^2 \rho(E_F) v_f^2 \tau_e (1 - \lambda k_F^2 + o(\lambda))$$

- New : Dependence of the diffusion constant on λ!

Quantum correction to conductivity

- Cooperon structure factor





- 1 singlet mode and 3 triplets : one single diffusive (gapless) mode

$$\Gamma^C_{s.s.}(\vec{Q}) = \frac{1}{DQ^2}|S\rangle\langle S|$$

$$\Gamma^C_{t.s.}(\vec{Q}) = \frac{1}{DQ^2+m_i}|T_i\rangle\langle T_i|$$

- Weak anti-localization expected!

Conclusions and perspectives

- Linear dependence in λ of the elastic scattering time
- Diffusion constant dependence in λ
- Weak anti-localization preserved

- 2nd order in λ for diffusion constant
- Derivation of the quantum correction to conductivity :
    - Characteristic mag. field
    - Extra contributions from triplet states of cooperon

Thanks for your attention!

Work supported by DFG grant
HA 5893/4-1 within SPP 1666