Transport of Dirac Fermions in Presence of Spin-orbit Impurities

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Outline

- Introduction to transport in 3D topological insulators
 - 3DTI surface states and transport
 - Regime of coherent transport (weak localization)
- Effects of spin-orbit impurities in 3DTI
 - Elastic scattering time
 - Diffusion constant
 - Quantum correction to conductivity



-0.1 3D Topological in Eigh

3DTI : insulator with odd r protected surface states (B



Tuned Bi2-0 Ca0Se3 Figure 2 | Transverse-momentum k, de energy of 21 eV (corresponding to 0^{3} k-BZ) are shown. Although the bands belo broad feature show weaker k_z dispersion dependence of the U-shaped continuer 15-31 eV photon energies reveal two dis-Dirac-cone bands inside the gap. <u>The Pi</u> also observed in BiSb; ref. 5). c, A k-space theta (θ) range of $\pm 30^{\circ}$. This map (k_z , k_z

-0.6

-0.2

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k,, (Å⁻¹)

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at particular high-symmetry points fulkrhandsponnaute high Ses and Bibles, is specific and surface BZ. In our calculations, the SSs (ee Meusulipet) are interested liver in the surface bar in the second second liver in the second surface BZ. In our calculations, the SSs (ecological surface BZ. In our calculations, the SSs (ecological surface BZ. In our calculations, the SSs (ecological surface BZ. In Signature Steph), the SSs (ecological surface such as gold 25,26 or 10 (ref. 5). In Bi₂Se₃, the SSs emerge from the bulk continuum each other at $\overline{\Gamma}$, pass through the Ferna Diappoint of the state of the second sta merge with the bulk conduction-band continuum@ensus.

Bi2 Te3 Alpichshey et al pair of Kramers points. Our lanchard design the no surface band crosses the Fermi level if SQC is not inc

- the calculation, and only with the inclusio Strong spin-orbit couplingd on Spin-morner tur the Fermi level. The calculated band topology with realistic SOC
- leads to a single ring-like surface FS, which Bring degenerate aventributions to a particular photoening Dirac fermions Hamilton is insident with the Z2=-1 class in marking and and some the West in the standard and some the standard classification scheme⁷.

A global agreement between the experience and a solution and a solution of the (Fig. 1a-c) and our theoretical calculation (Fig. 14) in obtained by on the instrumentation of the instrumentation

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19 eV

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Tuned Bi2-8Ca8Se3

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Figure 1 Detection of spin-momentum locking of

High

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 $k_x (\text{\AA}^{-1})^{\circ}$

Bi₂Te₃

Transport in mesoscopic physics

- Mesoscopic physics = weak disorder, coherent transport $\lambda_F \ll l_e \ll L, L_\phi$
- Scattering of the electrons on impurities
- Each trajectory has a given probability amplitude a_i
- Conductivity $\sigma \propto \sum a_i^* a_j$



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Quantum corrections to conductivity

In case of time-reversal symmetry : added contribution



• Constructive interference suppressed by magnetic field : Weak localization (Altshuler *et al.*, 1980)



Weak anti-localization

• In presence of spin orbit impurities (Hikami et al., 1980)

 $V(\vec{k},\vec{k}') = U(Id + i\lambda(\vec{k}\times\vec{k}').\vec{\sigma})$

• Elastic scattering time modification

$$\frac{1}{\tau_e} = 2\pi\rho(E_F)n_I U^2 (1 + \lambda^2 k_F^4)$$

• Quantum correction to conductivity : $\sigma = \sigma_{cl} - \frac{\alpha e^2}{\pi^2 \hbar} \ln L$

2 spin 1/2 : 4 cooperon modes

- 3 triplet (+1/2, killed)
- I singlet (-1/2, preserved)

 $\alpha : 1 \rightarrow -1/2$

Weak anti-localization!



impurities for a 2DEG with parabolic dispersion

Coherent transport of Dirac fermions

• Dirac fermions + scalar disorder : weak anti-localization



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Elastic scattering time

Model : Dirac fermions + weak scalar disorder + SOC from impurities

$$\mathcal{H} = \hbar v_F (\vec{k} \times \vec{\sigma})_z + V(\vec{k}, \vec{k}')$$
$$V(\vec{k}, \vec{k}') = U(Id + i\lambda(\vec{k} \times \vec{k}').\vec{\sigma})$$

• Elastic scattering time via Fermi golden rule

$$\frac{1}{\tau_e} = \pi \rho(E_F) n_I U^2 \left(1 + \lambda k_F^2 + \frac{\lambda^2 k_F^4}{2} \right)$$

• Self energy calculation

$$\frac{1}{\tau_e} = \pi \rho(E_F) n_I U^2 \left(1 + \lambda k_F^2 + \frac{\lambda^2 k_F^4}{2} \right)$$

• New : Linear dependance in λ of the elastic scattering time!

Diffusion constant $\sigma = e^2 \rho(E_F) D$

• Solving the kinetic equation

$$-e\vec{E}.\vec{\nabla}_{\vec{k}}f = \int \frac{d\vec{k'}}{(2\pi)^2} 2\pi |\langle \vec{k'}|V|\vec{k}\rangle|^2 \delta(E(\vec{k'}) - E(\vec{k}))(f(\vec{k'}) - f(\vec{k}))$$

$$\sigma_{cl} = e^2 \rho(E_F) v_f^2 \tau_e (1 - \lambda k_F^2 + o(\lambda))$$

• Standard diagrammatic technique (ladder diagram)



• New : Dependence of the diffusion constant on λ !

Quantum correction to conductivity

• Cooperon structure factor



 I singlet mode and 3 triplets : one single diffusive (gapless) mode

$$\Gamma_{s.s.}^{C}(\vec{Q}) = \frac{1}{DQ^{2}} |S\rangle \langle S|$$

$$\Gamma_{t.s.}^{C}(\vec{Q}) = \frac{1}{DQ^{2} + m_{i}} |T_{i}\rangle \langle T_{i}|$$

• Weak anti-localization expected!

Conclusions and perspectives

- Linear dependence in λ of the elastic scattering time
- Diffusion constant dependence in λ
- Weak anti-localization preserved

- 2nd order in λ for diffusion constant
- Derivation of the quantum correction to conductivity :
 - Characteristic mag. field
 - Extra contributions from triplet states of cooperon

Thanks for your attention!

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