## Weak Antilocalization of 3DTI Surface States in the Presence of Spin-orbit Impurities

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#### Outline

- Introduction to transport in 3D topological insulators
  - 3DTI surface states and transport
  - Regime of coherent transport (weak localization)
- Effects of spin-orbit impurities in 3DTI
  - Elastic scattering time
  - Diffusion constant
  - Quantum correction to conductivity



#### -0.1 3D Topological in Eigh

3DTI : insulator with odd r protected surface states (B

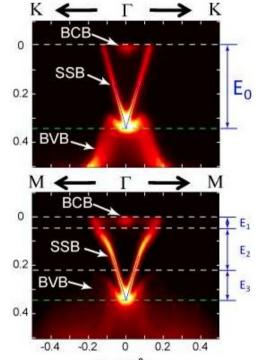


Figure 2 | Transverse-momentum k, de energy of 21 eV (corresponding to  $0^{3}$  k-BZ) are shown. Although the bands belo broad feature show weaker  $k_z$  dispersion dependence of the U-shaped continuer 15-31 eV photon energies reveal two dis-Dirac-cone bands inside the gap. <u>The Pi</u> also observed in BiSb; ref. 5). c, A k-space theta ( $\theta$ ) range of  $\pm 30^{\circ}$ . This map ( $k_z$ ,  $k_z$ 

-0.6

-0.2

0

k,, (Å<sup>−1</sup>)

0.2

at particular high-symmetry points fulkrhandsponnaute high Ses and Bibles, is specific and surface BZ. In our calculations, the SSs (ee Meusulipet) are interested live by the surface by surface BZ. In our calculations, the SSs (ecological surface BZ. In our calculations, the SSs (ecological surface BZ. In our calculations, the SSs (ecological surface BZ. In Signature Stephene), the surface such as gold  $^{25,26}$  or  $^{10}$  (ref. 5). In Bi<sub>2</sub>Se<sub>3</sub>, the SSs emerge from the bulk continuum each other at  $\overline{\Gamma}$ , pass through the Ferna Diappoint of the state of the second sec merge with the bulk conduction-band continuum@ensus.

Bi2 Te3 Alpichshever a pair of Kramers points. Our lanchard design the k (1/Å) no surface band crosses the Fermi level if SQC is not inc

- the calculation, and only with the inclusio Strong spin-orbit couplingd on Spin-morner tur the Fermi level. The calculated band topology with realistic SOC
- Dirac fermions Hamilton is insident with the Z2=-1 class in marking and and some the West in the standard of the standard in t classification scheme<sup>7</sup>.

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19 eV

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0

k,, (Å⁻¹)

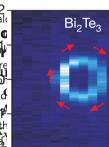
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 $k_x (\text{\AA}^{-1})^{\circ}$ 

-0.1

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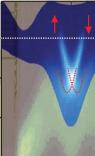
-0.4 0

 $E_{\rm B}$  (eV)



0.1

High

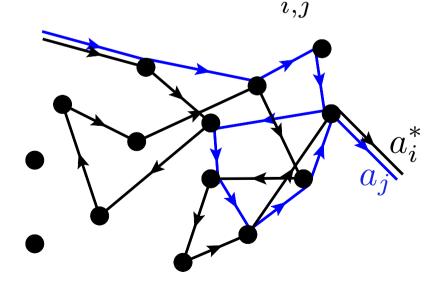


mienduyo o v o An acpetien so the boland / (Fig. 2c), allowing for a way to distin leads to a single ring-like surface FS, which Bring degenerate aventributions to a particular photoenings Figure 1 Detection of spin-momentum locking of

A global agreement between the experience and a solution and a solution of the (Fig. 1a-c) and our theoretical calculation (Fig. 14) in obtained by on the instrumentation of the instrumentation

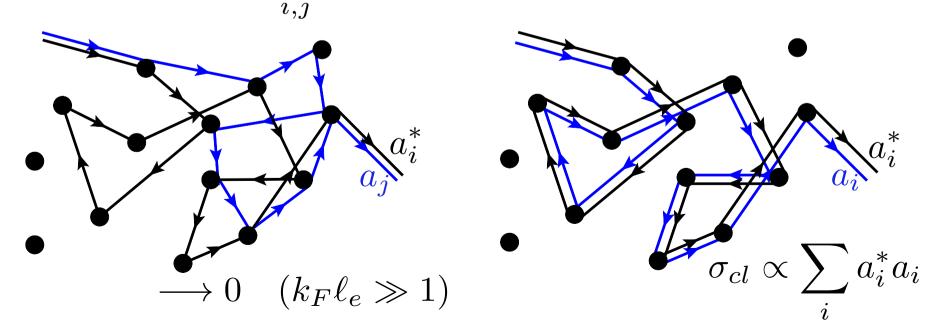
#### Transport in mesoscopic physics

- Mesoscopic physics = weak disorder, coherent transport  $\lambda_F \ll l_e \ll L, L_\phi$
- Scattering of the electrons on impurities
- Each trajectory has a given probability amplitude  $a_i$
- Conductivity  $\sigma \propto \sum a_i^* a_j$



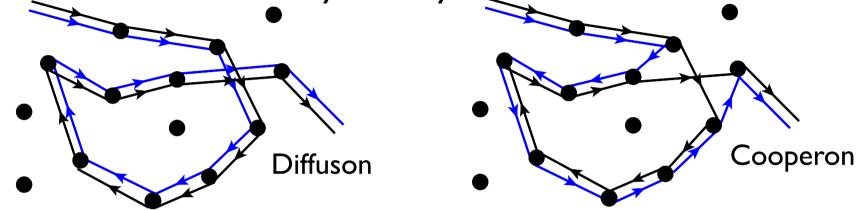
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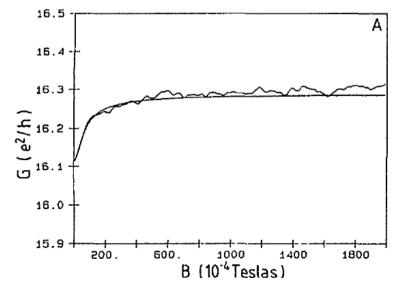


#### Quantum corrections to conductivity

In case of time-reversal symmetry : added contribution



• Constructive interference suppressed by magnetic field : Weak localization (Altshuler *et al.*, 1980)



#### Weak anti-localization

• In presence of 3D spin orbit impurities (Hikami et al., 1980)

$$V(\vec{k}, \vec{k}') = U(Id + i\lambda(\vec{k} \times \vec{k}').\vec{\sigma})$$

• Elastic scattering time modification

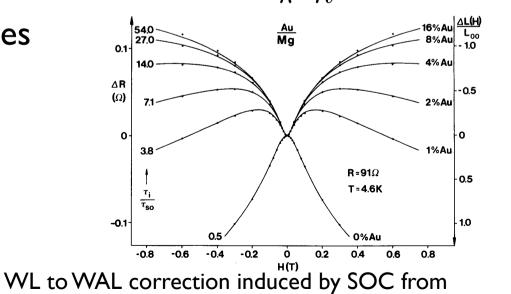
$$\frac{1}{\tau_e} = 2\pi\rho(E_F)n_I U^2 (1 + \lambda^2 k_F^4)$$

• Quantum correction to conductivity :  $\sigma = \sigma_{cl} - \frac{\alpha e^2}{\pi^2 \hbar} \ln L$ 

- 2 spin 1/2 : 4 cooperon modes
- 3 triplet (+1/2, killed)
- I singlet (-1/2, preserved)

 $\alpha : 1 \rightarrow -1/2$ 

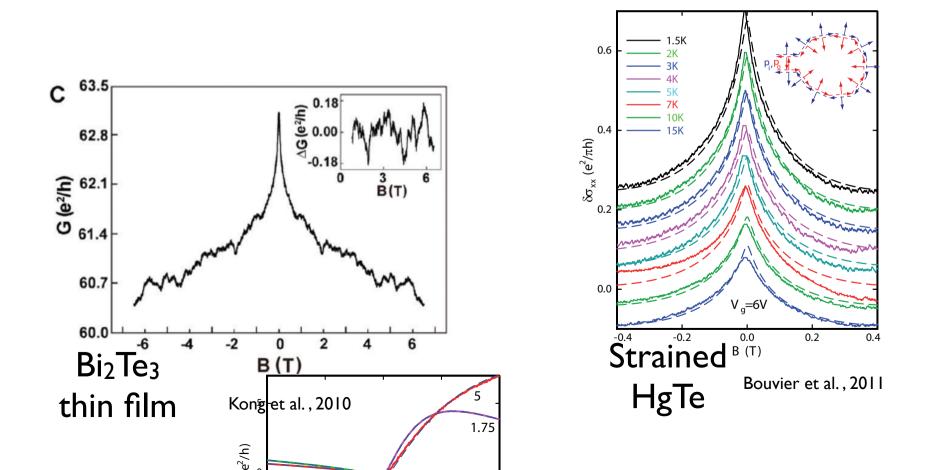
Weak anti-localization!



impurities for electrons with parabolic dispersion

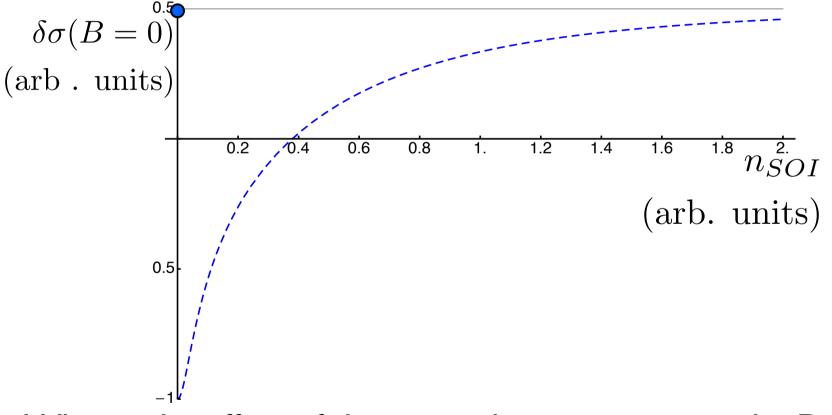
#### Coherent transport of Dirac fermions

 Dirac fermions + scalar disorder : weak anti-localization th :Tkachov and Hankiewicz, PRB 84 (2011) Adroguer et al., NJP 14 (2012)



#### Summary of coherent transport

- Dirac fermions + scalar disorder : weak anti-localization (dot)
- Electrons w/ parabolic dispersion + 3D spin-orbit impurities : crossover from weak localization to weak anti-localization (line)



• What is the effect of the spin-orbit impurities on the Dirac surface states physics?

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#### Elastic scattering time

Model : Dirac fermions + weak scalar disorder + SOC from impurities

$$\mathcal{H} = \hbar v_F (\vec{k} \times \vec{\sigma})_z + V(\vec{k}, \vec{k}')$$
$$V(\vec{k}, \vec{k}') = U(Id + i\lambda(\vec{k} \times \vec{k}').\vec{\sigma})$$

• Elastic scattering time via Fermi golden rule

$$\frac{1}{\tau_e} = \pi \rho(E_F) n_I U^2 \left( 1 + \lambda k_F^2 + \frac{\lambda^2 k_F^4}{2} \right)$$

• Self energy calculation

$$\frac{1}{\tau_e} = \pi \rho(E_F) n_I U^2 \left( 1 + \lambda k_F^2 + \frac{\lambda^2 k_F^4}{2} \right)$$

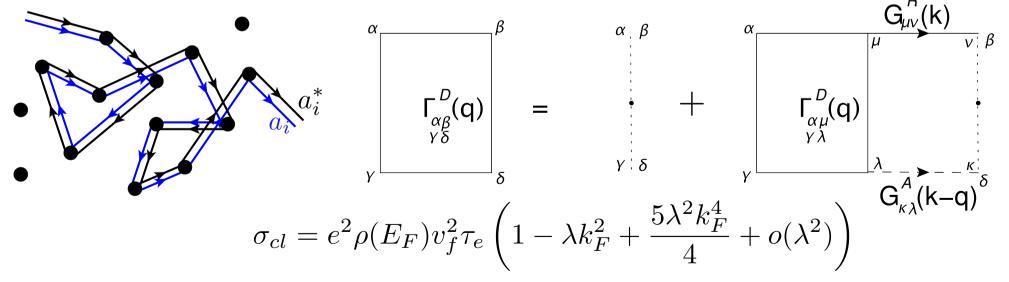
• New : Linear dependance in  $\lambda$  of the elastic scattering time!

#### **Diffusion constant** $\sigma = e^2 \rho(E_F) D$

• Solving the kinetic equation

$$e\vec{E}.\vec{\nabla}_{\vec{k}}f = \int \frac{d\vec{k'}}{(2\pi)^2} 2\pi |\langle \vec{k'}|V|\vec{k}\rangle|^2 \delta(E(\vec{k'}) - E(\vec{k}))(f(\vec{k'}) - f(\vec{k}))$$
  
$$\sigma_{cl} = e^2 \rho(E_F) v_f^2 \tau_e \left(1 - \lambda k_F^2 + \frac{5\lambda^2 k_F^4}{4} + o(\lambda^2)\right)$$

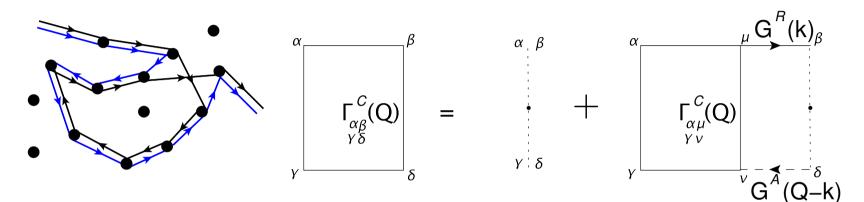
• Standard diagrammatic technique (ladder diagram)



• New : Dependence of the diffusion constant on  $\lambda$ !

### Quantum correction to conductivity

• Cooperon structure factor

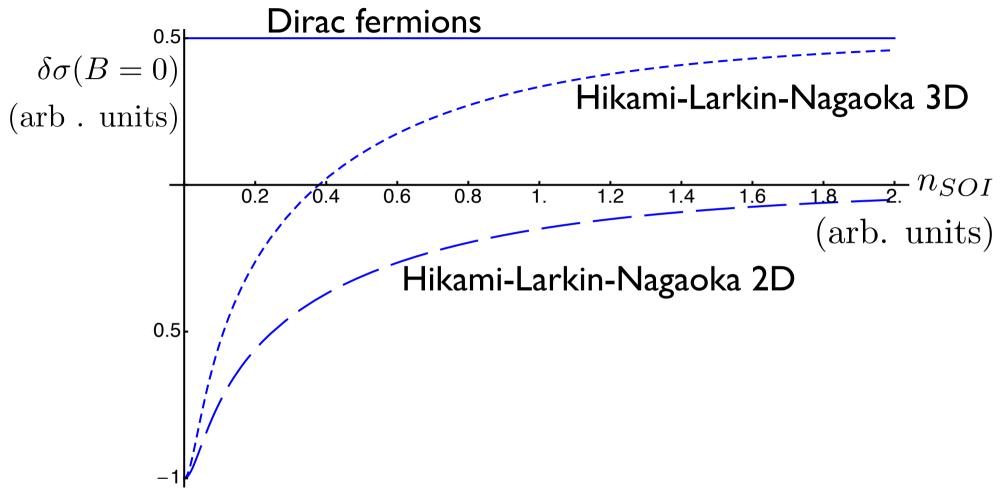


 I singlet mode and 3 triplets : one single diffusive (gapless) mode

$$\Gamma_{s.s.}^{C}(\vec{Q}) = \frac{1}{DQ^{2}} |S\rangle \langle S|$$
  
$$\Gamma_{t.s.}^{C}(\vec{Q}) = \frac{1}{DQ^{2} + m_{i}} |T_{i}\rangle \langle T_{i}|$$

Always weak anti-localization

#### What we learnt in coherent transport?



- Dirac fermions : always weak antilocalization
- Hikami-Larkin-Nagaoka : dependance on the dimension (in 2D, SU(2) symmetry not totally broken, I triplet remains)

#### Conclusions and perspectives

- Linear dependence in  $\lambda$  of the elastic scattering time
- Diffusion constant dependence in  $\lambda$
- Weak anti-localization preserved

- Hikami-Larkin-Nagaoka formula do not give WAL for surface states
- Derivation of the quantum correction to conductivity in presence of magnetic field
  - Characteristic mag. field

# Thanks for your attention!

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