

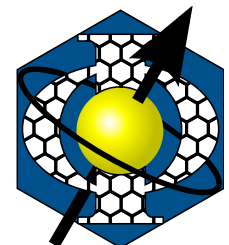
# Weak Antilocalization of 3DTI Surface States in the Presence of Spin-orbit Impurities

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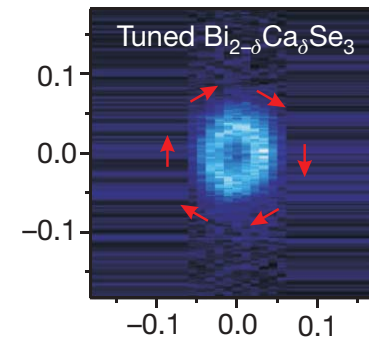
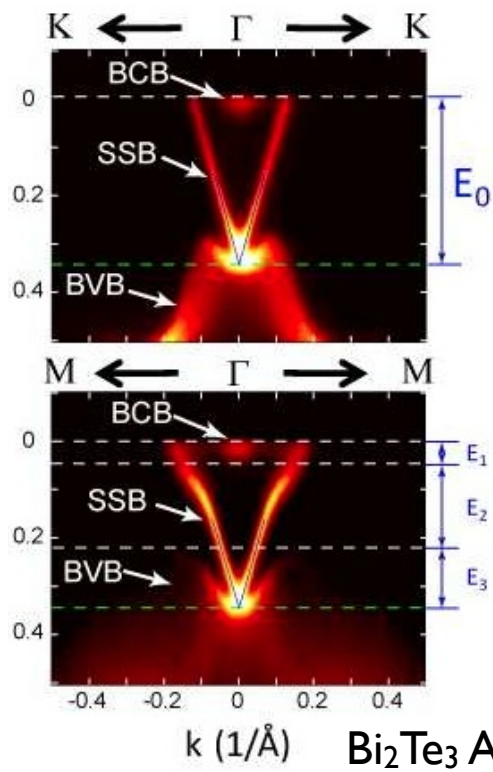


# Outline

- Introduction to transport in 3D topological insulators
  - 3DTI surface states and transport
  - Regime of coherent transport (weak localization)
- Effects of spin-orbit impurities in 3DTI
  - Elastic scattering time
  - Diffusion constant
  - Quantum correction to conductivity
- Perspectives

# 3D Topological insulators surface states

- 3DTI : insulator with odd number of topologically protected surface states ( $\text{Bi}_2\text{Te}_3$  ,  $\text{Bi}_2\text{Se}_3$ , strained  $\text{HgTe}$ ... )



D. Hsieh *et al.*, 2009

- Strong spin-orbit coupling : Spin-momentum locking
- Dirac fermions Hamiltonian :  $\mathcal{H} = \hbar v_F (\vec{k} \times \vec{\sigma})_z$

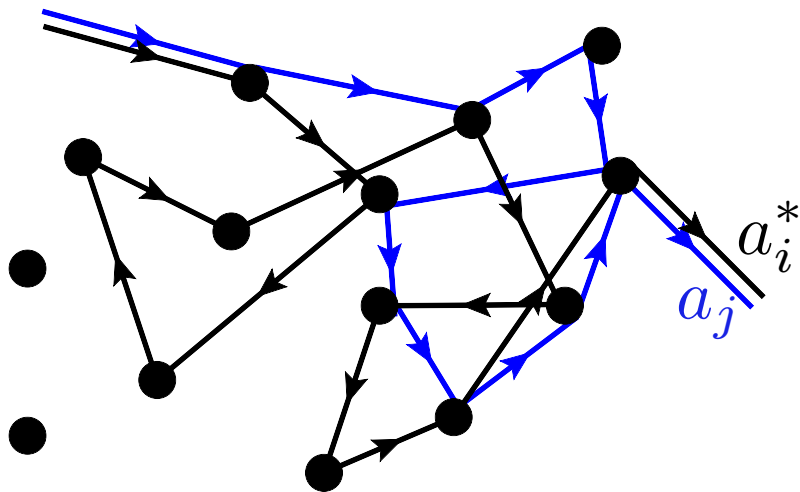
# Transport in mesoscopic physics

- Mesoscopic physics = weak disorder, coherent transport

$$\lambda_F \ll l_e \ll L, L_\phi$$

- Scattering of the electrons on impurities
- Each trajectory has a given probability amplitude  $a_i$

- Conductivity  $\sigma \propto \sum_{i,j} a_i^* a_j$



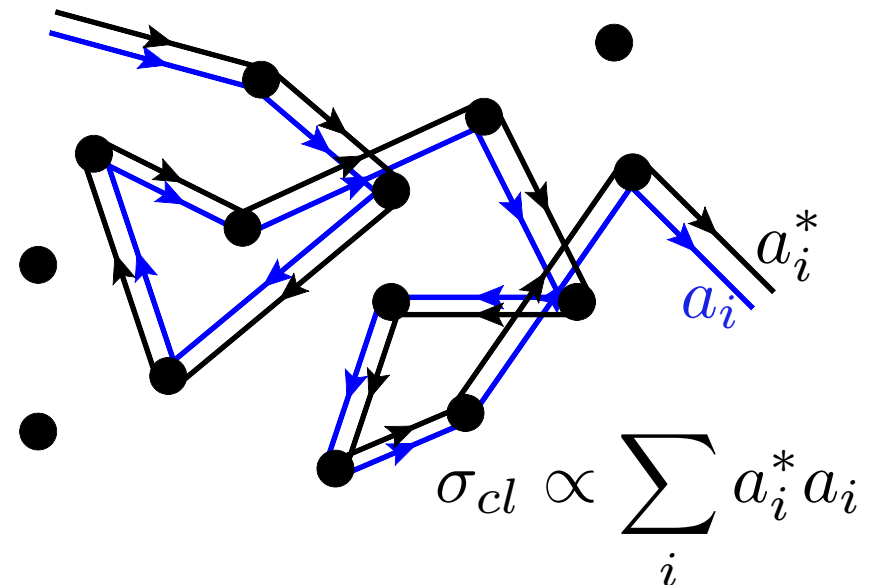
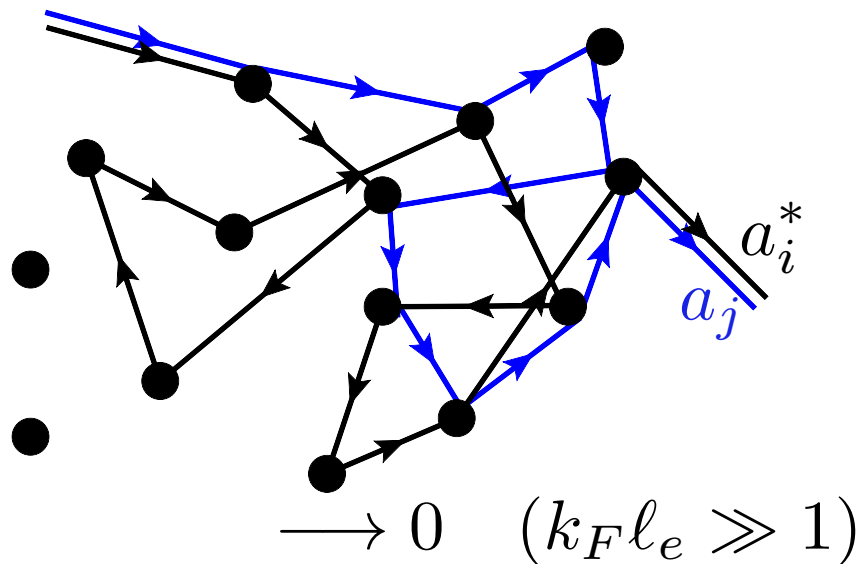
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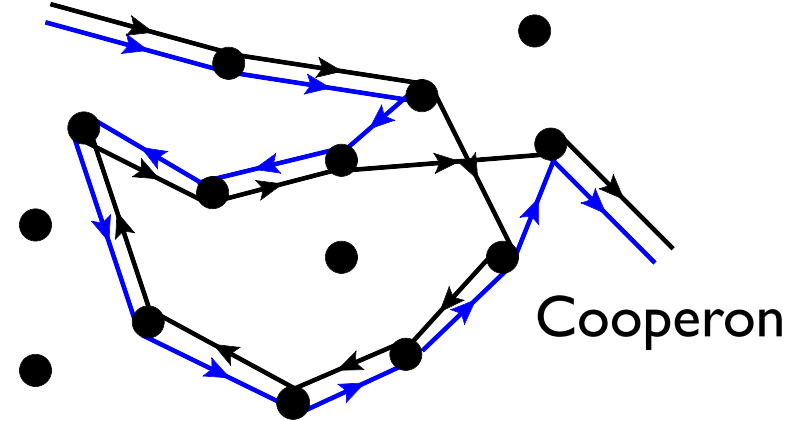
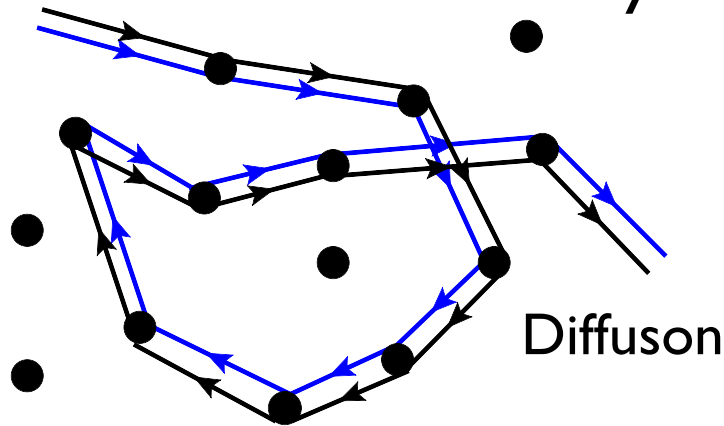
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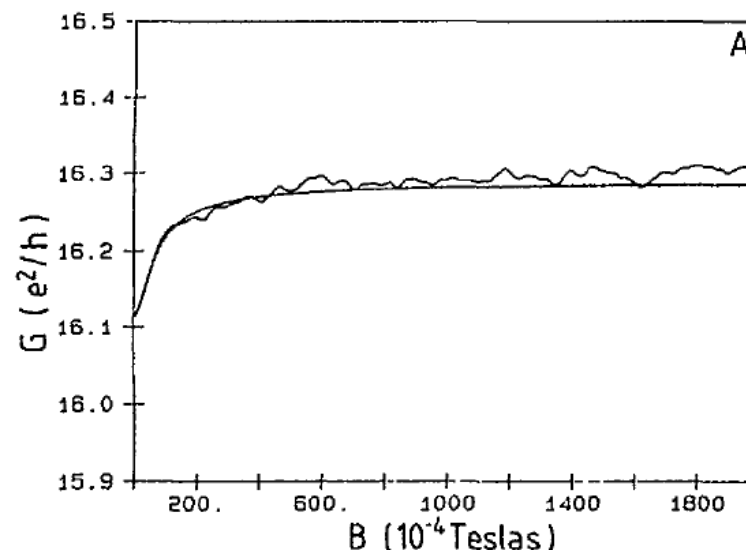


# Quantum corrections to conductivity

- In case of time-reversal symmetry : added contribution



- Constructive interference suppressed by magnetic field : Weak localization (Altshuler *et al.*, 1980)



# Weak anti-localization

- In presence of 3D spin orbit impurities (Hikami *et al.*, 1980)

$$V(\vec{k}, \vec{k}') = U(Id + i\lambda(\vec{k} \times \vec{k}') \cdot \vec{\sigma})$$

- Elastic scattering time modification

$$\frac{1}{\tau_e} = 2\pi\rho(E_F)n_I U^2 (1 + \lambda^2 k_F^4)$$

- Quantum correction to conductivity :  $\sigma = \sigma_{cl} - \frac{\alpha e^2}{\pi^2 \hbar} \ln L$

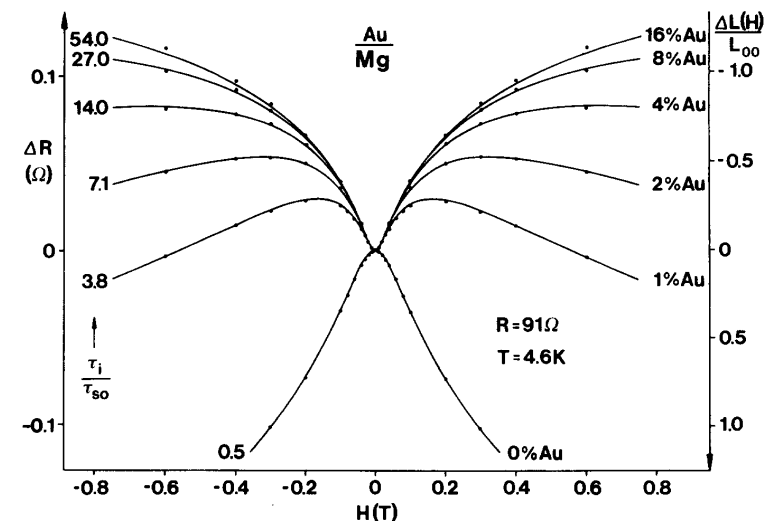
2 spin 1/2 : 4 cooperon modes

- 3 triplet (+1/2, killed)

- 1 singlet (-1/2, preserved)

$$\alpha : 1 \rightarrow -1/2$$

**Weak anti-localization!**



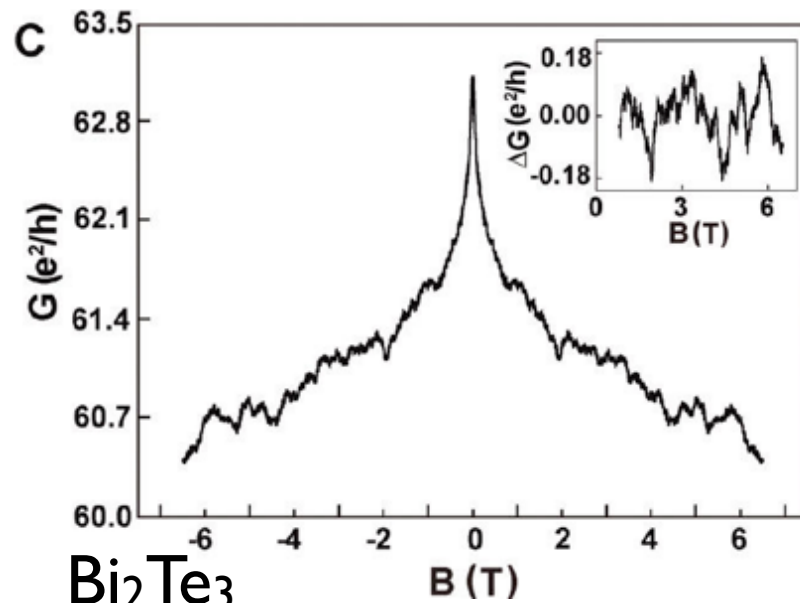
WL to WAL correction induced by SOC from impurities for electrons with parabolic dispersion

# Coherent transport of Dirac fermions

- Dirac fermions + scalar disorder : weak anti-localization

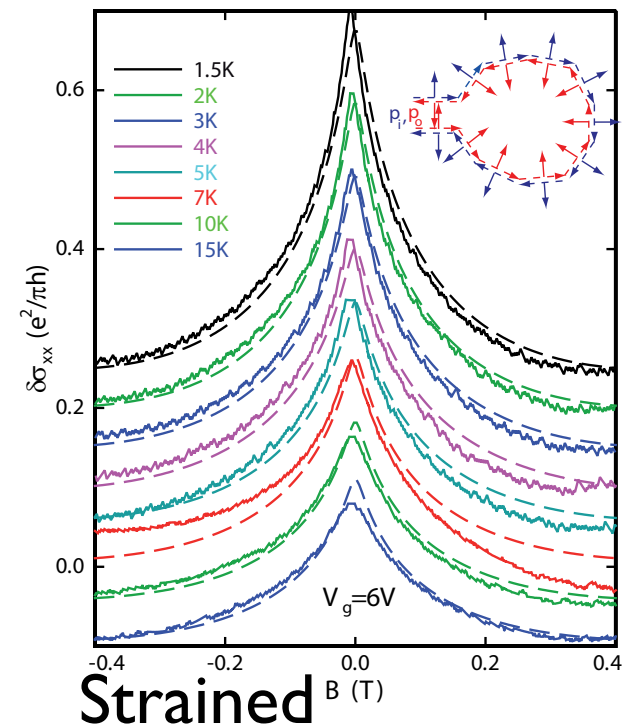
th :Tkachov and Hankiewicz, PRB 84 (2011)

Adroguer et al., NJP 14 (2012)



$\text{Bi}_2\text{Te}_3$   
thin film

Kong et al., 2010



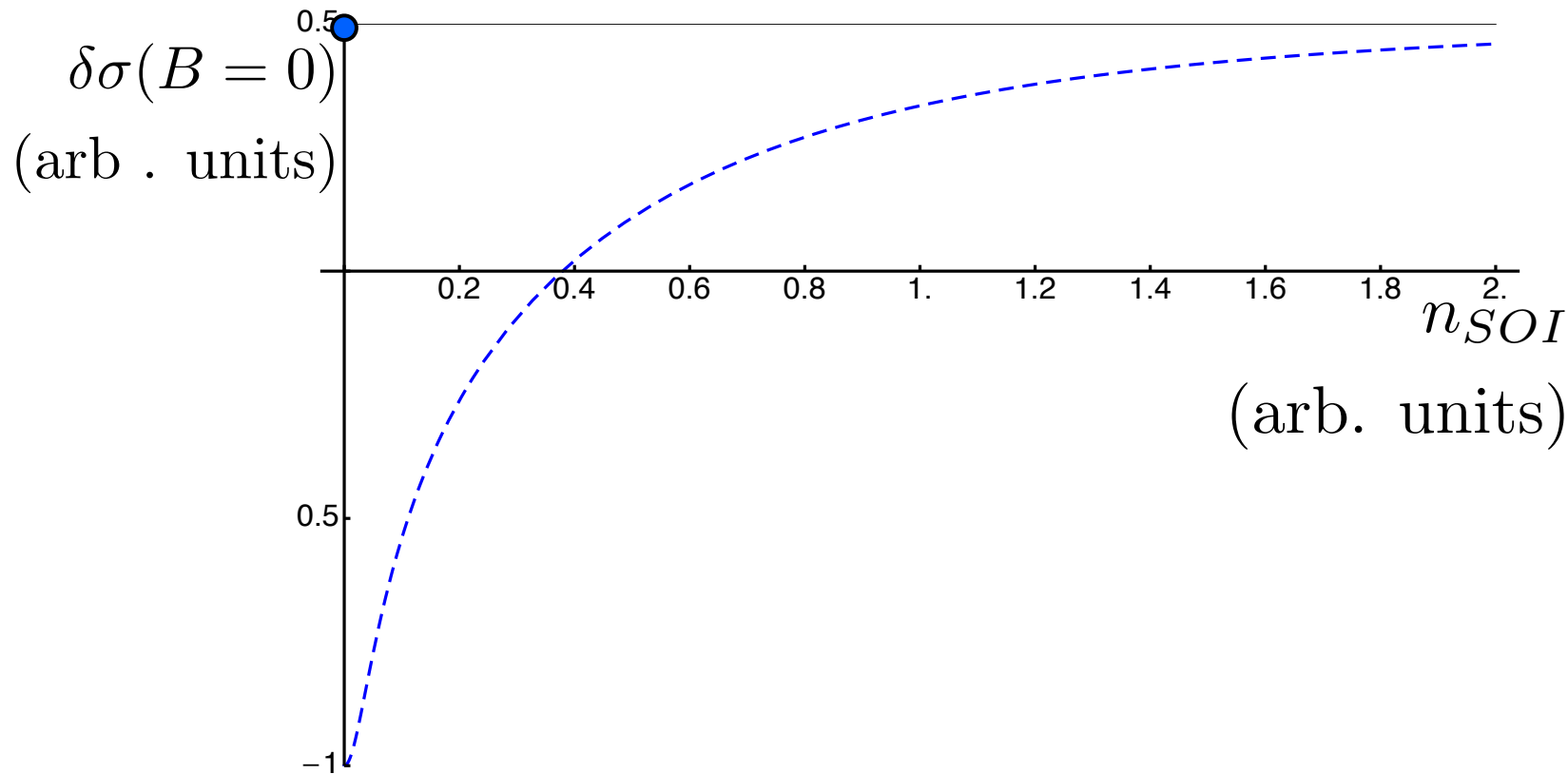
Strained  
 $\text{HgTe}$

Bouvier et al., 2011



# Summary of coherent transport

- Dirac fermions + scalar disorder : weak anti-localization (dot)
- Electrons w/ parabolic dispersion + 3D spin-orbit impurities : crossover from weak localization to weak anti-localization (line)



- What is the effect of the spin-orbit impurities on the Dirac surface states physics?

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# Elastic scattering time

- Model : Dirac fermions + weak scalar disorder + SOC from impurities

$$\mathcal{H} = \hbar v_F (\vec{k} \times \vec{\sigma})_z + V(\vec{k}, \vec{k}')$$
$$V(\vec{k}, \vec{k}') = U (Id + i\lambda (\vec{k} \times \vec{k}') \cdot \vec{\sigma})$$

- Elastic scattering time via Fermi golden rule

$$\frac{1}{\tau_e} = \pi \rho(E_F) n_I U^2 \left( 1 + \lambda k_F^2 + \frac{\lambda^2 k_F^4}{2} \right)$$

- Self energy calculation

$$\frac{1}{\tau_e} = \pi \rho(E_F) n_I U^2 \left( 1 + \lambda k_F^2 + \frac{\lambda^2 k_F^4}{2} \right)$$

- New : Linear dependance in  $\lambda$  of the elastic scattering time!

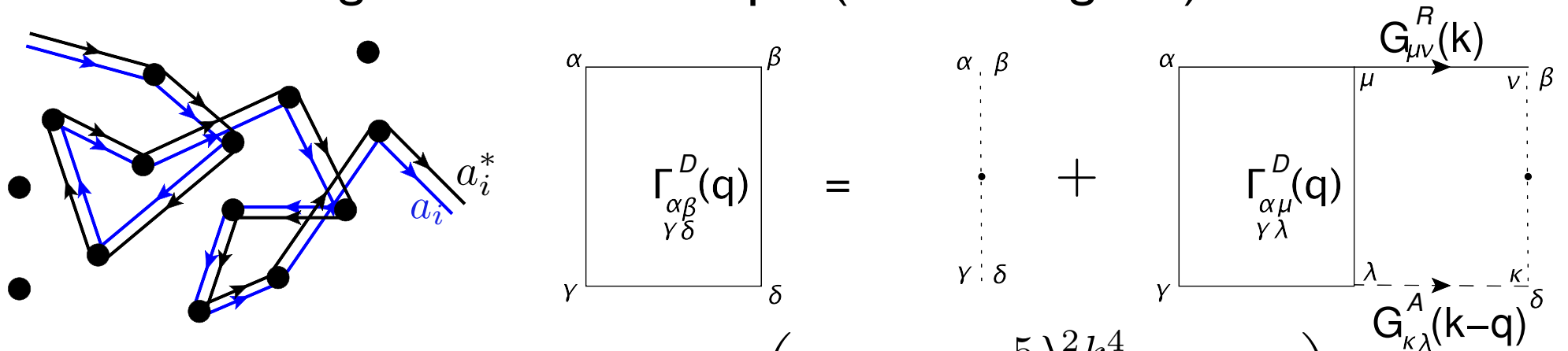
# Diffusion constant $\sigma = e^2 \rho(E_F) D$

- Solving the kinetic equation

$$-e\vec{E} \cdot \vec{\nabla}_{\vec{k}} f = \int \frac{d\vec{k}'}{(2\pi)^2} 2\pi |\langle \vec{k}' | V | \vec{k} \rangle|^2 \delta(E(\vec{k}') - E(\vec{k})) (f(\vec{k}') - f(\vec{k}))$$

$$\sigma_{cl} = e^2 \rho(E_F) v_f^2 \tau_e \left( 1 - \lambda k_F^2 + \frac{5\lambda^2 k_F^4}{4} + o(\lambda^2) \right)$$

- Standard diagrammatic technique (ladder diagram)

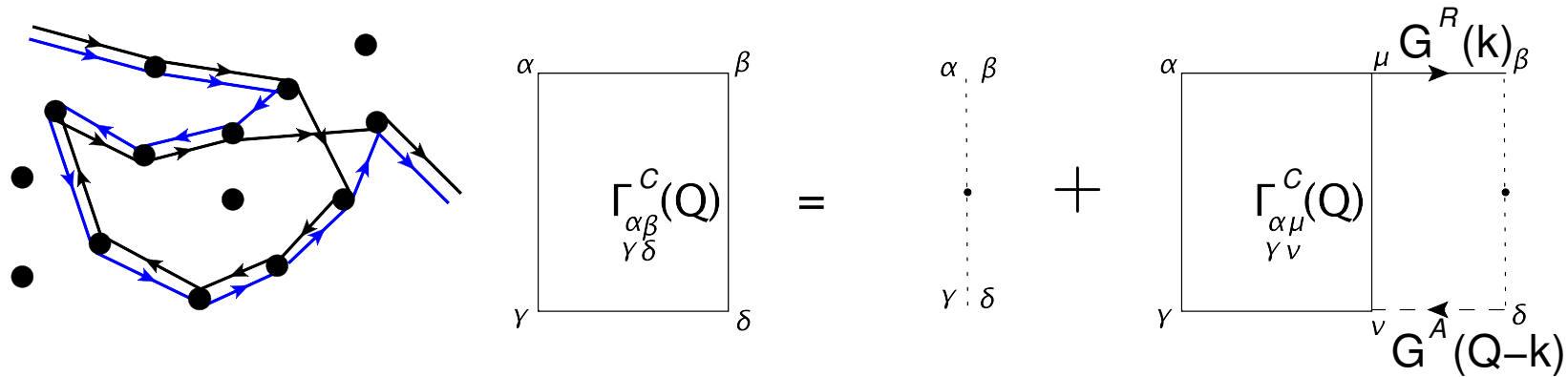


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- New : Dependence of the diffusion constant on  $\lambda$ !

# Quantum correction to conductivity

- Cooperon structure factor



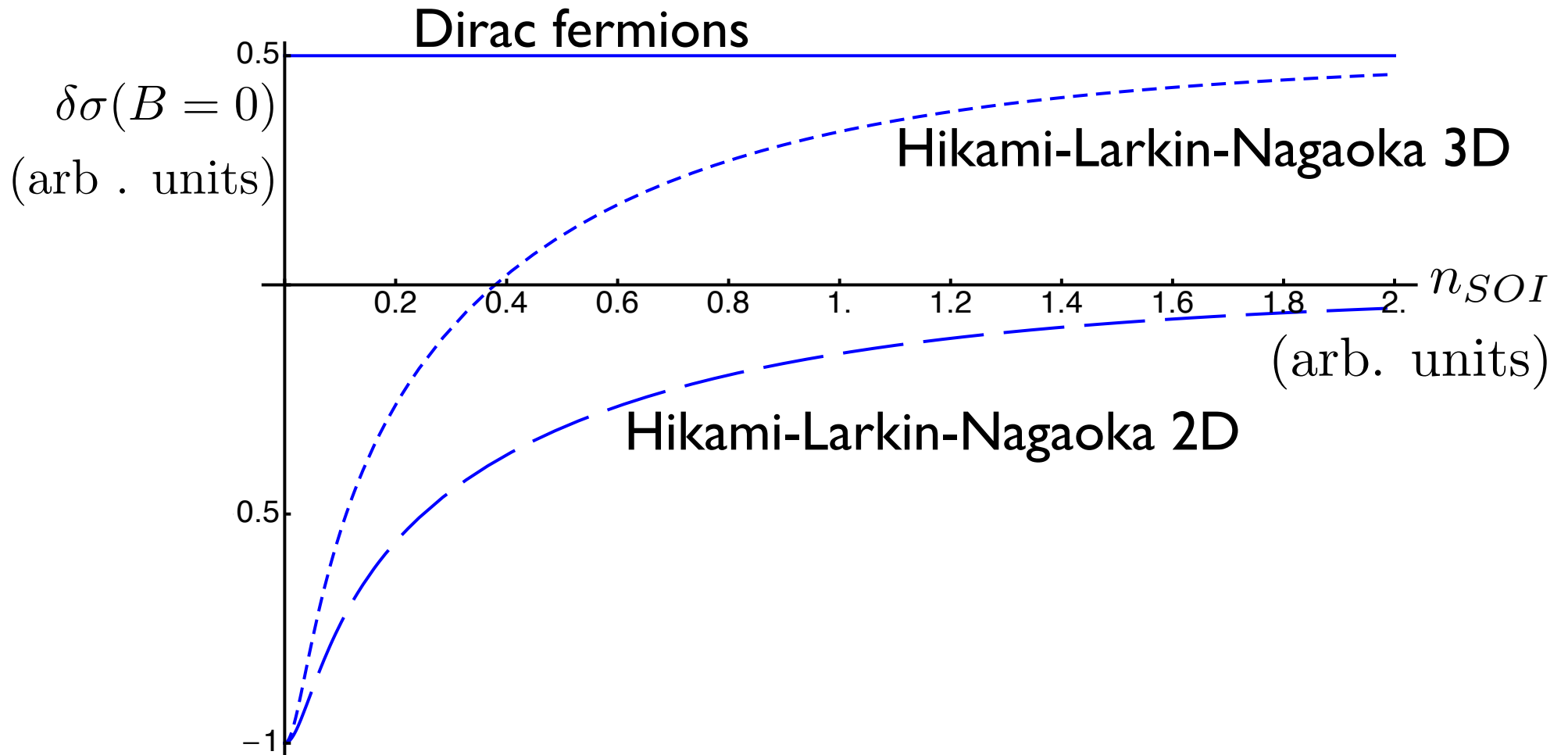
- 1 singlet mode and 3 triplets : one single diffusive (gapless) mode

$$\Gamma_{s.s.}^C(\vec{Q}) = \frac{1}{DQ^2} |S\rangle\langle S|$$

$$\Gamma_{t.s.}^C(\vec{Q}) = \frac{1}{DQ^2 + m_i} |T_i\rangle\langle T_i|$$

- Always weak anti-localization

# What we learnt in coherent transport?



- Dirac fermions : always weak antilocalization
- Hikami-Larkin-Nagaoka : dependance on the dimension (in 2D, SU(2) symmetry not totally broken, 1 triplet remains)

# Conclusions and perspectives

- Linear dependence in  $\lambda$  of the elastic scattering time
- Diffusion constant dependence in  $\lambda$
- Weak anti-localization preserved
- Hikami-Larkin-Nagaoka formula do not give WAL for surface states
- Derivation of the quantum correction to conductivity in presence of magnetic field
  - Characteristic mag. field

**Thanks for your  
attention!**

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