Diffusion at the surface of Topological Insulators

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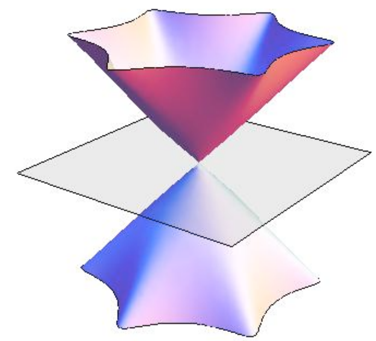
ENS DE LYON

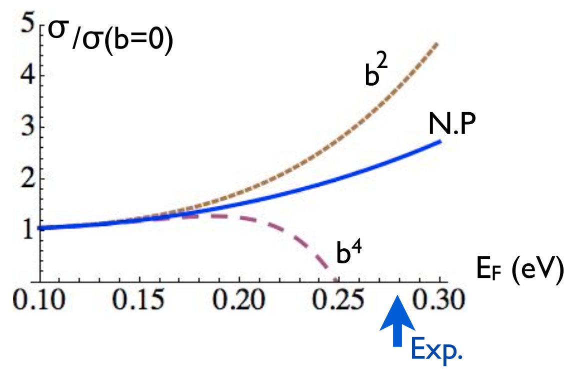
http://arxiv.org/abs/1205.5209

Outline

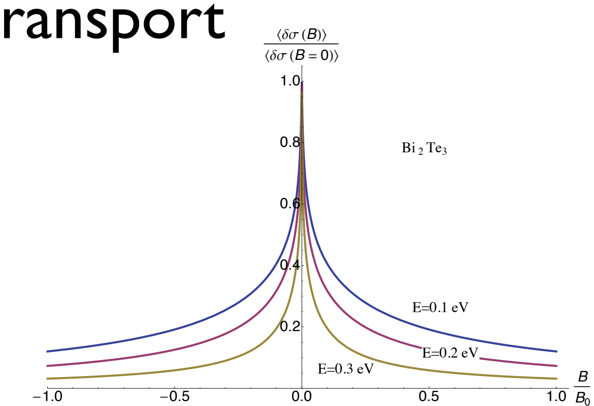
- Topological Insulators surface states



- Classical conductance

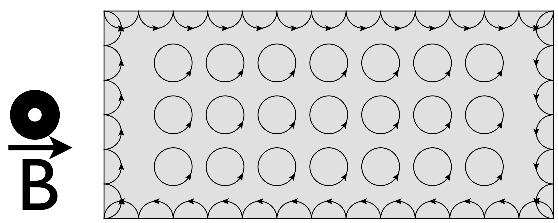


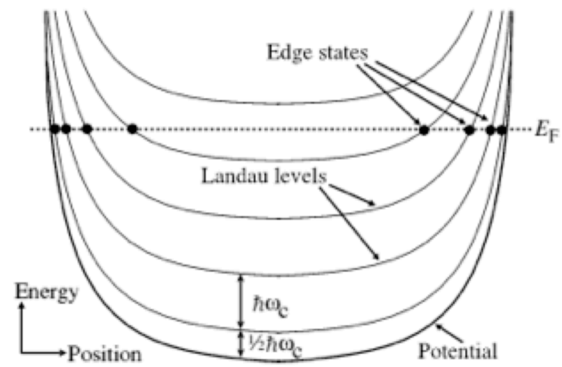
- Coherent transport



Topological insulators

- Insulating bulk with robust conducting surface states : Quantum Hall effect ?



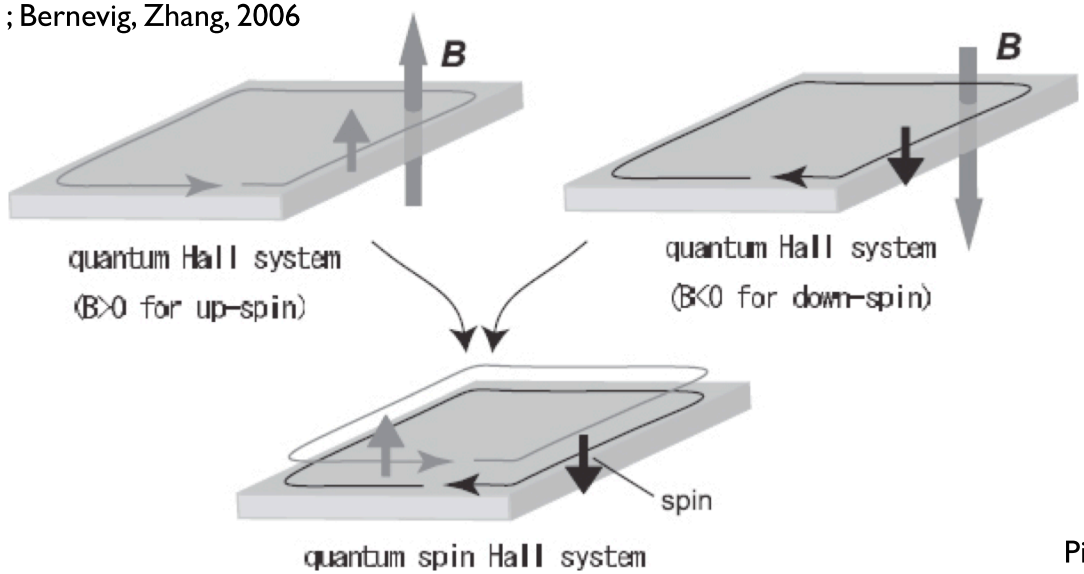


- Paradigm : 2D + Time Reversal symmetry breaking

Topological insulators

- Paradigm : 2D + Time reversal symmetry breaking
- 2D + Time-reversal symmetry : Spin orbit coupling Vs. magnetic field

Kane, Mele 2005 ; Bernevig, Zhang, 2006



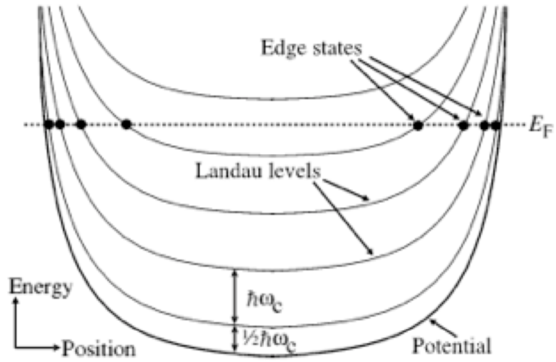
Pict. : Murakami

- 3D + TRS : topological insulators ; realized with $Bi_{1-x}Sb_x$, Bi_2Te_3 , Bi_2Se_3 , Strained HgTe , etc

Topological insulators surface states

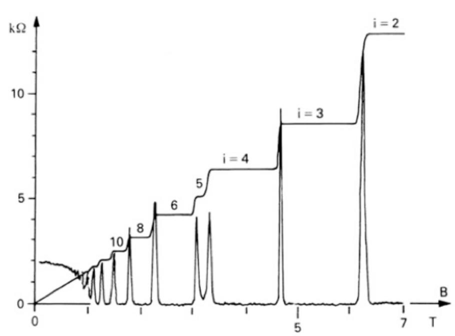
Quantum Hall Effect

- Robust edge states



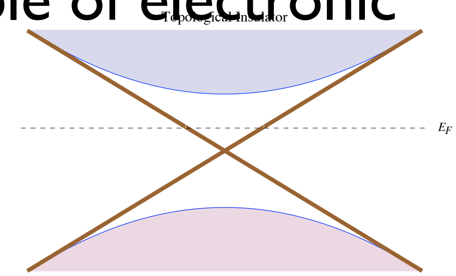
- Responsible of electronic transport

Buttiker, 1982

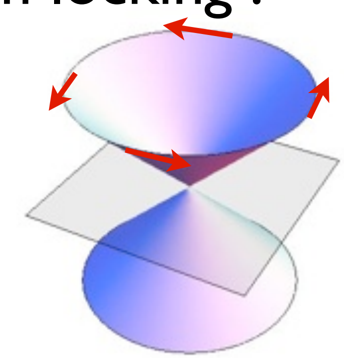


3D TI

- Robust surface states (odd number)
- Responsible of electronic transport

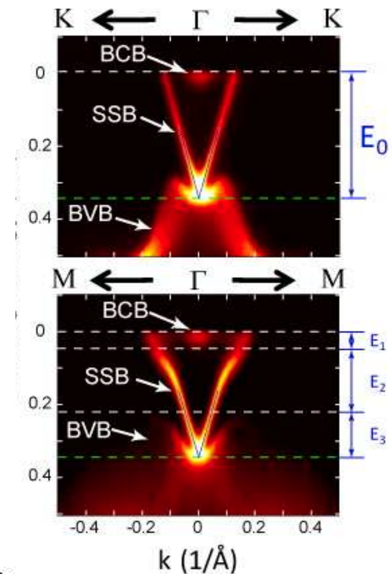


- Linear dispersion + momentum-spin locking : Dirac fermions

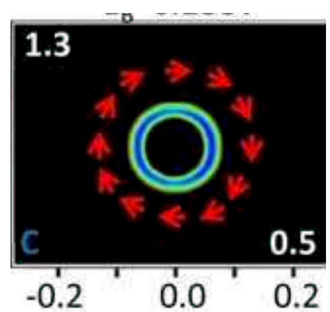


Transport of these surface states

ARPES data for the surface states

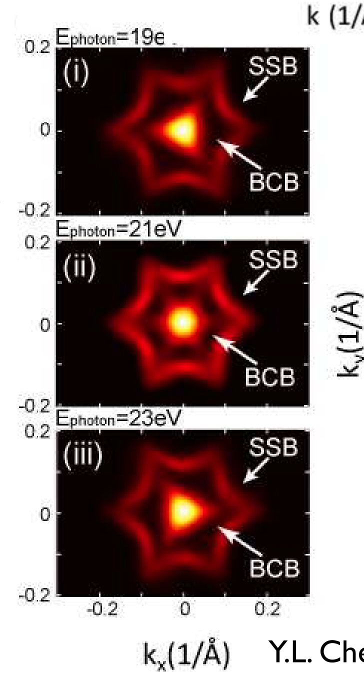


- Dirac fermions : linear dispersion
- <u>Magnetic</u> spin in the plane, winding around vertical axis



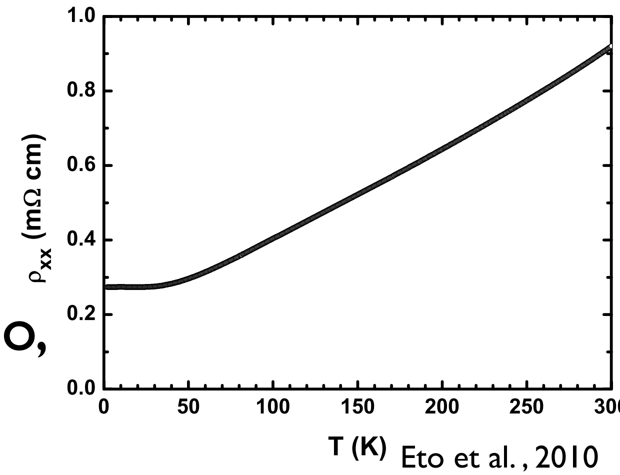
S.Y. Xu *et al.* (2011)

- Richer structure, hexagonal shape of the Fermi surface in Bi_2Te_3 and Bi_2Se_3



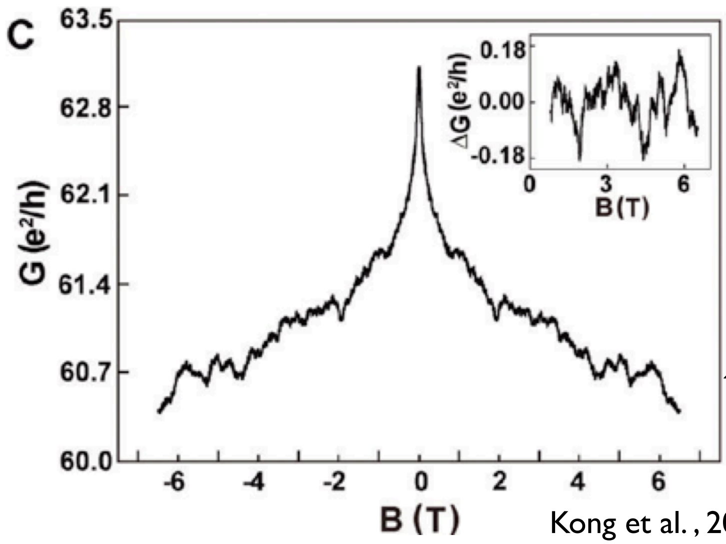
Y.L. Chen et al. , 2009

Transport experiments



Eto et al. , 2010

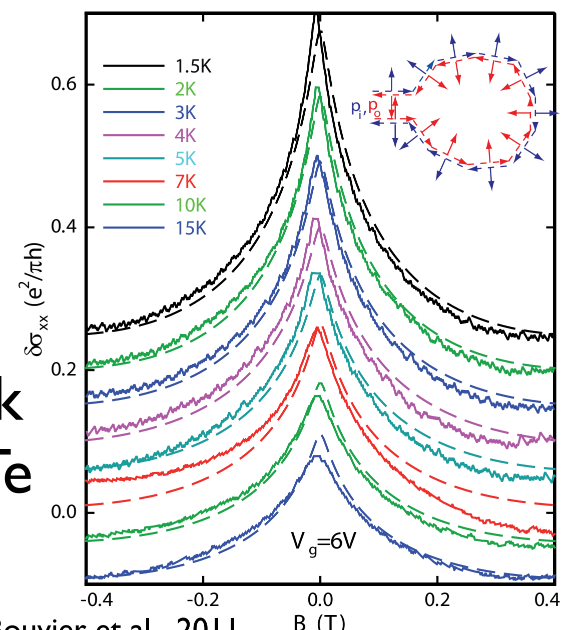
- Residual bulk conductance
  - Thin films : improve surface/bulk ratio, gating both surfaces
  - Strained HgTe : no bulk conductance
- Magneto-transport : weak anti-localization



Bi_2Te_3 thin film

Kong et al. , 2010

Bulk HgTe

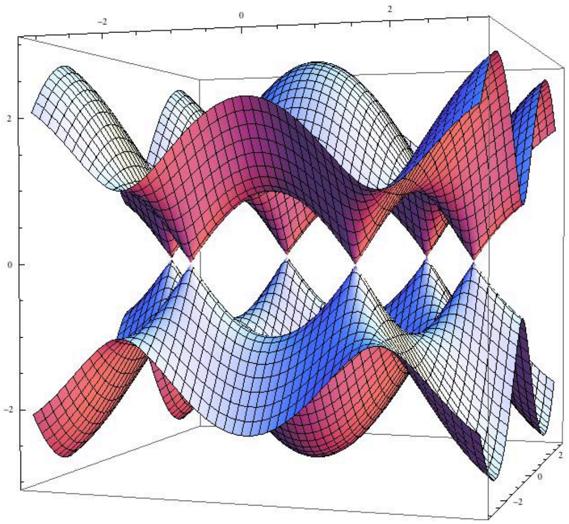


Bouvier et al. , 2011

Dirac fermions system $\mathcal{H} = \hbar v_f (\vec{\sigma} \times \vec{k}).\hat{z}$

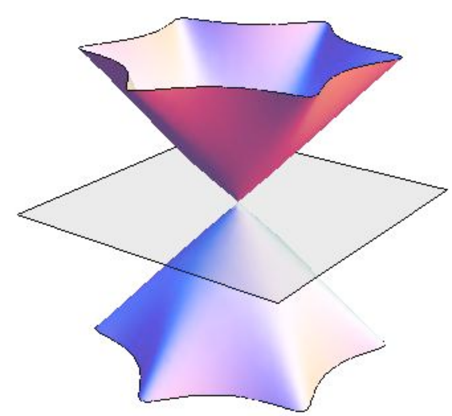
Graphene

- σ : sublattice
- 2 x 2 cones
- TRS : no constraint
- Trigonal warping at high energies



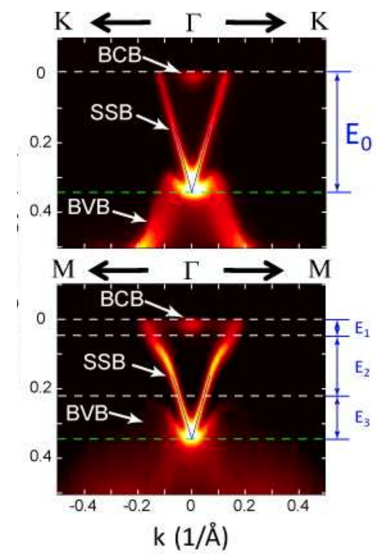
TI surface state

- σ : magnetic spin
- 1 cone (odd)
- TRS : constraint
- Hexagonal warping at high energies

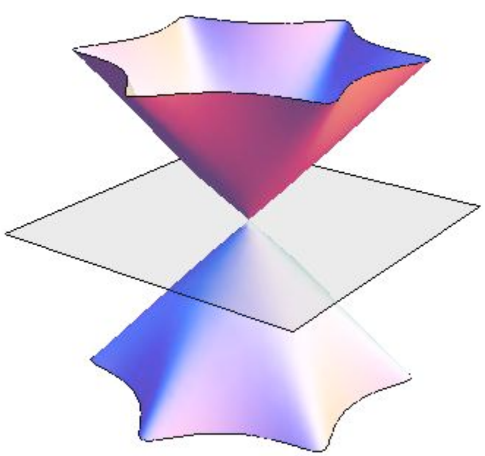


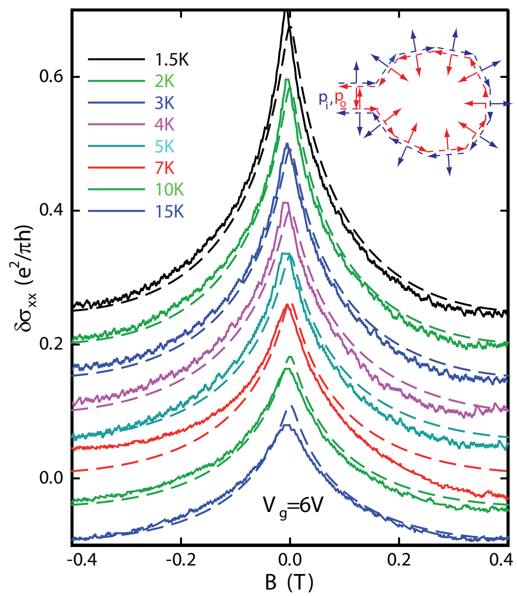
Departure from Dirac fermions

- Dirac point burried in bulk valence band
- High energy regime natural



- Hexagonal warping : effect on transport





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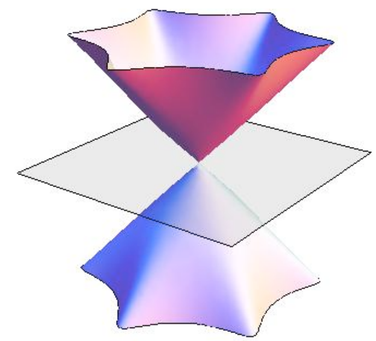

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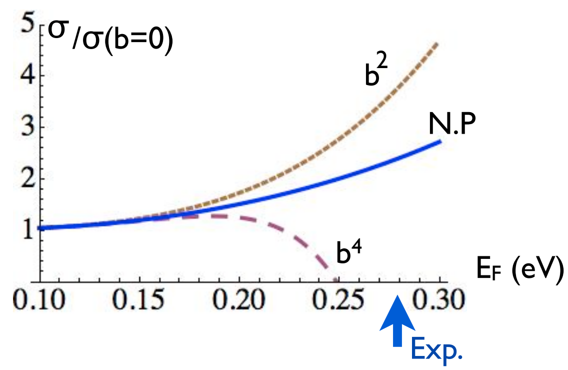
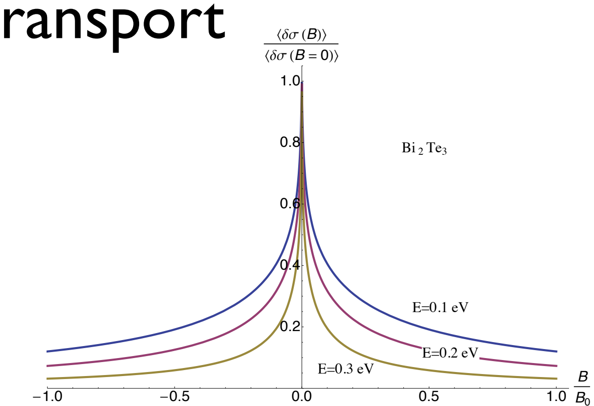
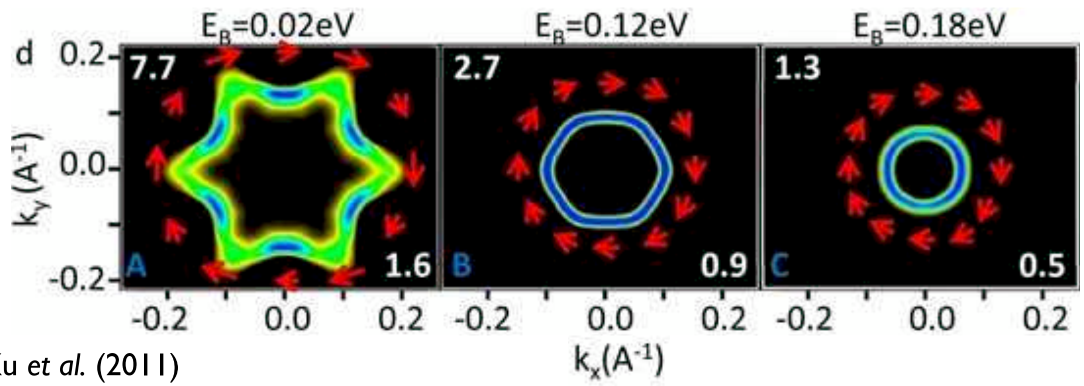


- Coherent transport



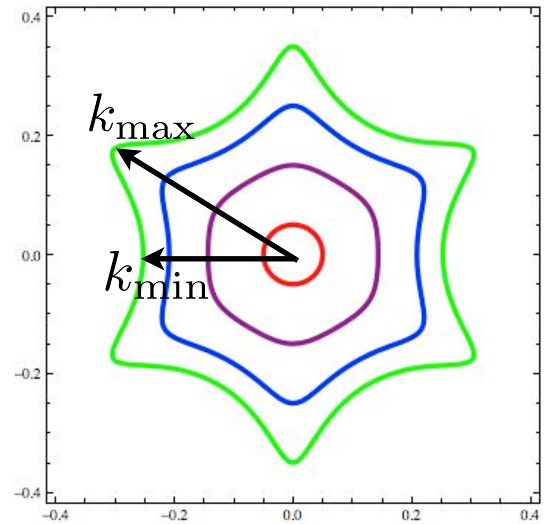
Model

- Fermi surface deformation



S.Y. Xu *et al.* (2011)

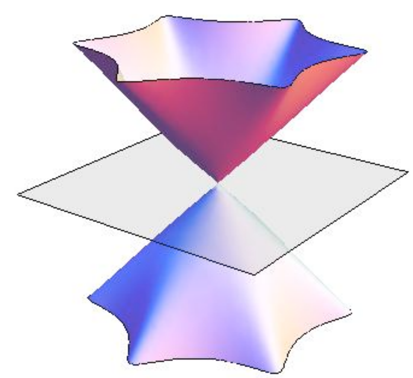
Different energies Fermi surfaces



- Warping hamiltonian

$$\mathcal{H} = \hbar v_F(\vec{\sigma} \times \vec{k}).\hat{z} + \frac{\lambda}{2}(k_+^3 + k_-^3)\sigma^z$$

(L. Fu, 2009)



$$b = \frac{\lambda E_F^2}{2(\hbar v_F)^3} = \frac{w(w + w_{\max})^2}{2(w_{\max} - w)^3} \quad ; \quad w = w_{\max}\frac{k_{\max} - k_{\min}}{k_{\max} + k_{\min}},$$

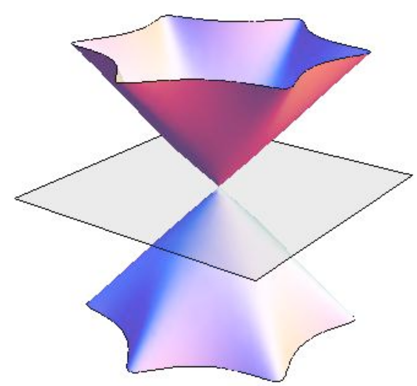
Experimentally : $0 \leqslant b \lesssim 0.6$

Regime of diffusive transport

- Experimental regime : far from the Dirac point (good metal)
- Hamiltonian : $\mathcal{H} = \hbar v_F (\vec{\sigma} \times \vec{k}).\hat{z} + \frac{\lambda}{2}(k_+^3 + k_-^3)\sigma^z + V(\vec{r})$

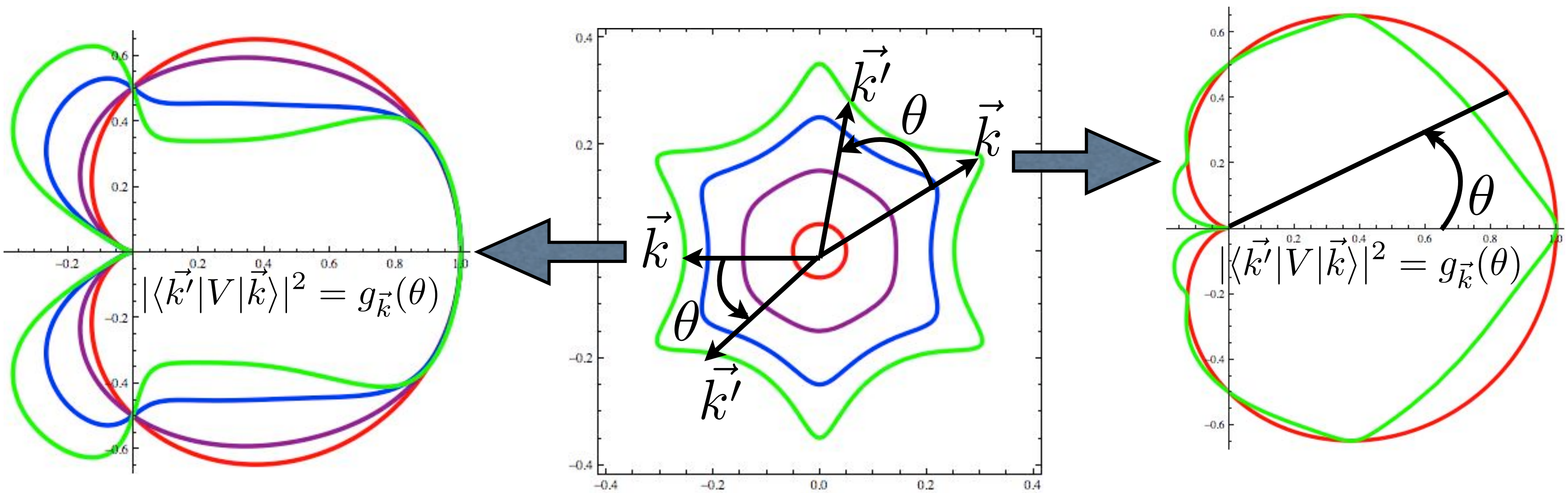
$$\langle V(\vec{r}) \rangle = 0 \qquad \langle V(\vec{r}) V(\vec{r'}) \rangle = \gamma \delta(\vec{r} - \vec{r'})$$

- Sample length $\gg$ mean free path ℓ_e (weak disorder)
- Semi classical approach, $k_f \ell_e \gg 1$ (perturbative approach)
  - Boltzmann equation
  - Diagrammatics



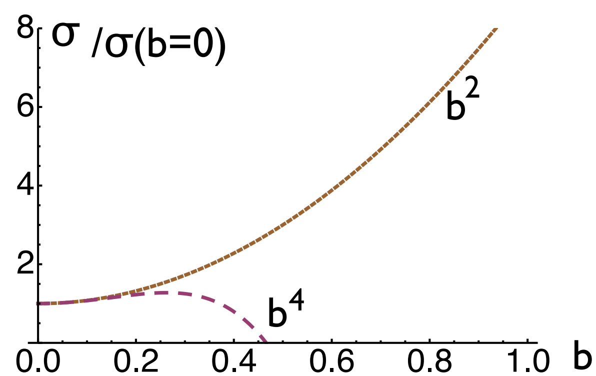
Boltzmann approach

- Density of states : $f(\vec{k})$
- Scattering probability : $|\langle\vec{k'}|V|\vec{k}\rangle|^2 = g_{\vec{k}}(\theta)$ spinor overlap



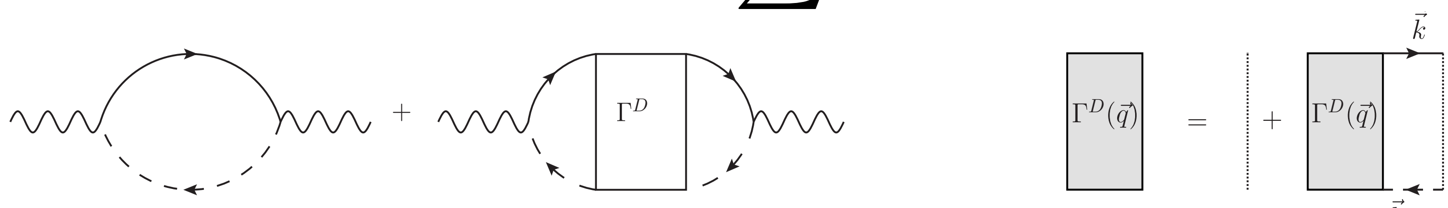
<u>Perturbative result</u>

$$\sigma = \frac{e^2}{h}\frac{2\hbar^2 v_F^2}{\gamma}(1 + 8b^2 - 58b^4 + o(b^4))$$

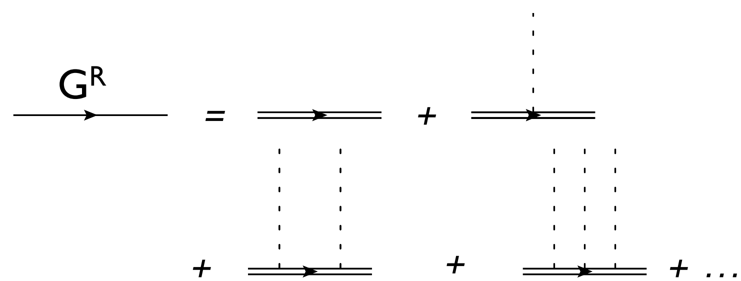


Diagrammatic approach

- Kubo formula : $\sigma_{\alpha\beta} \propto \sum j_\alpha G^R j_\beta G^A$

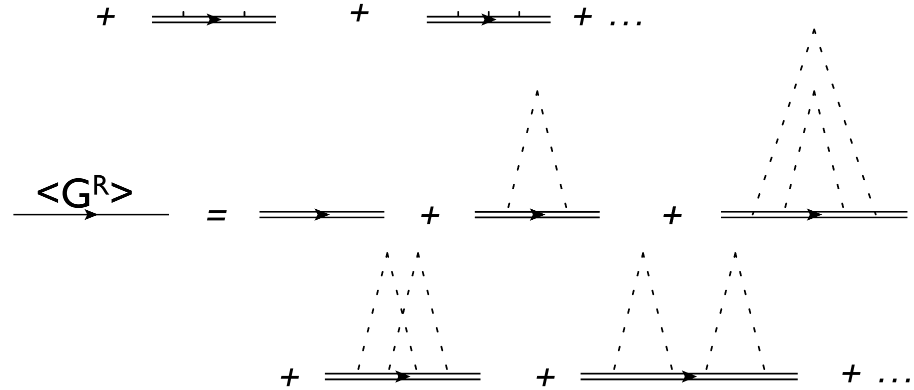


- Dyson equation :



$$\langle V(\vec{r})\rangle = 0$$

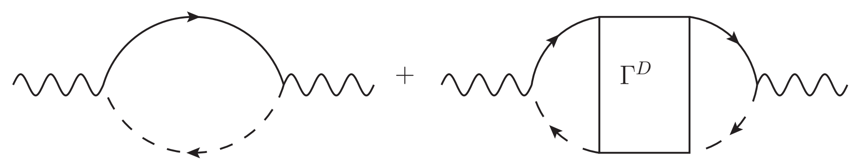
$$\langle V(\vec{r})V(\vec{r'})\rangle = \gamma\delta(\vec{r}-\vec{r'})$$



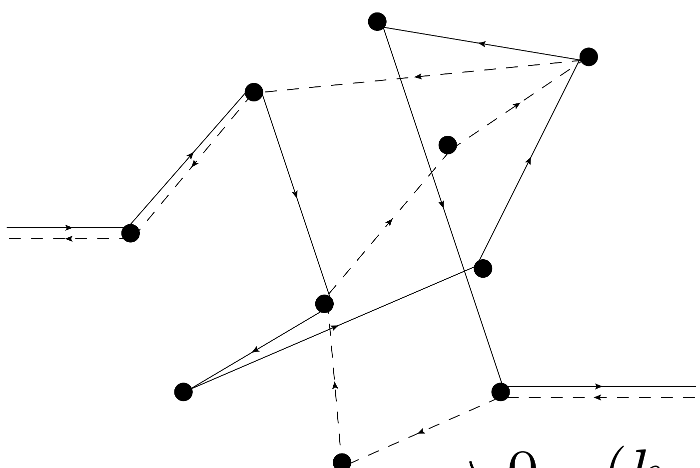
$$\boxed{\langle G^{R/A}\rangle = (E \pm i\hbar/2\tau_e - \mathcal{H}_0)^{-1}}$$

Diagrammatic approach

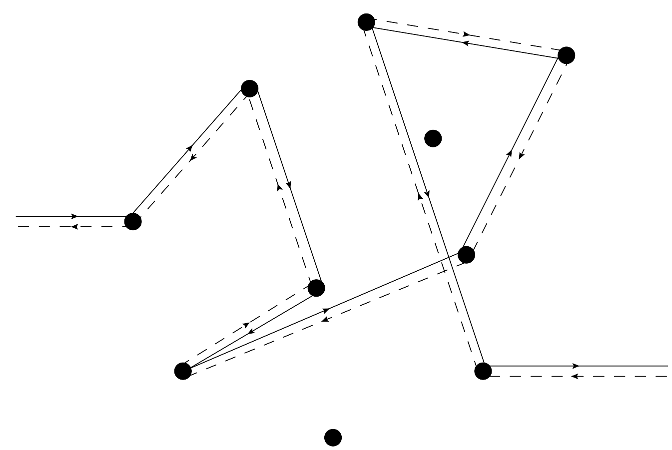
- Kubo formula : $\sigma_{\alpha\beta} \propto \sum j_\alpha G^R j_\beta G^A$



- Disorder induced coupling $\langle G^R G^A \rangle$

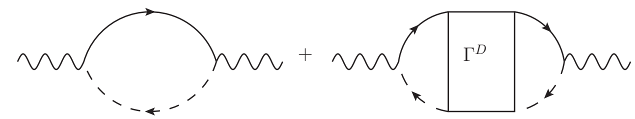


$\longrightarrow 0 \quad (k_F \ell_e \gg 1)$

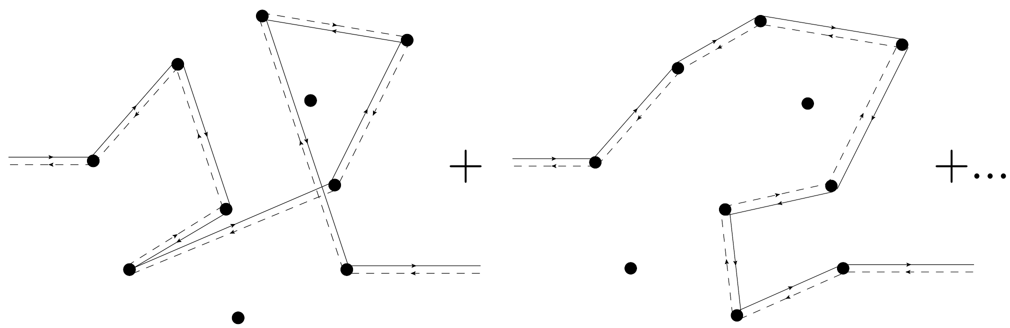


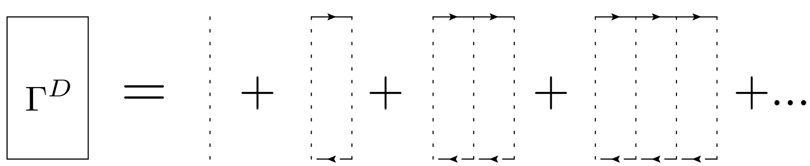
Diagrammatic approach

$$\sigma_{\alpha\beta} \propto \sum j_\alpha G^R j_\beta G^A$$



- Diffuson mode $\langle G^R G^A \rangle$





4 modes

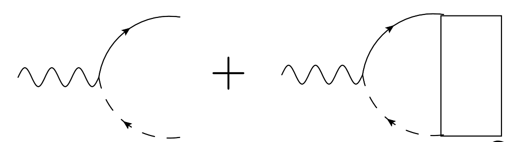
- 1 diffusive singlet $\dfrac{1}{D(b)q^2\tau_e - i\omega\tau_e/\hbar}$

- 3 triplet, non diffusive

Anisotropy

$$D = \frac{v_F^2 \tau_e}{d} \longrightarrow \frac{v_F^2 \tau_{tr}}{2}$$

- $b = 0$

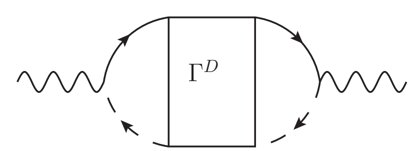


$$= -2ev_F\sigma^x$$

$\tau_{tr} = 2\ \tau_e$

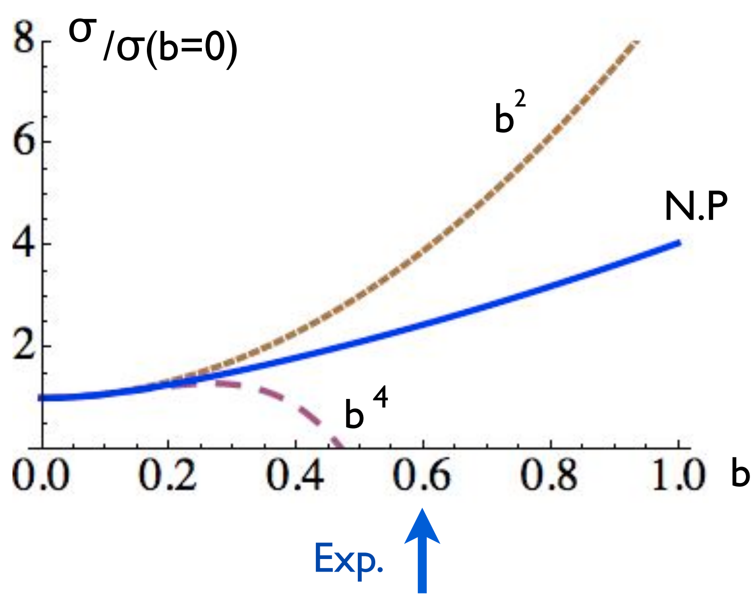
- $b \neq 0$

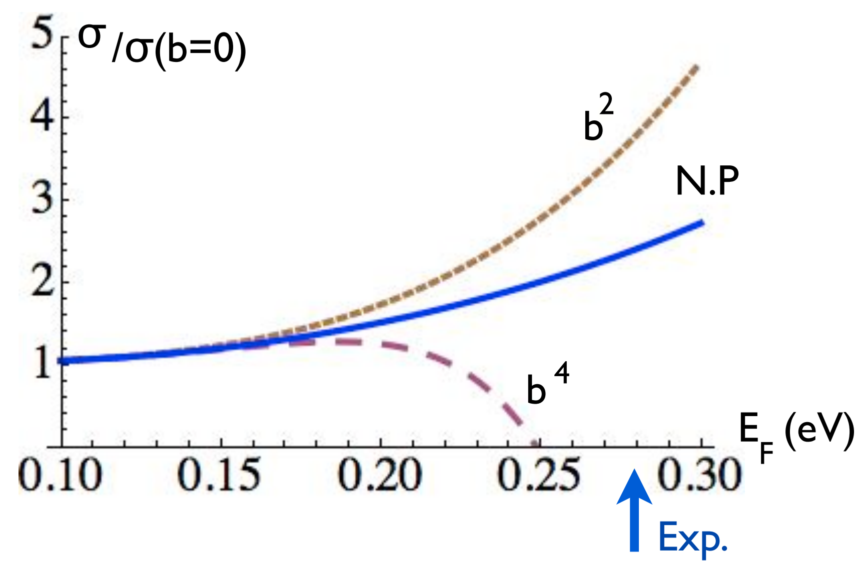
$$j_x = -ev_F(\sigma^x + 2bk^3 \cos 2\theta \sigma^z)$$



Non perturbative results

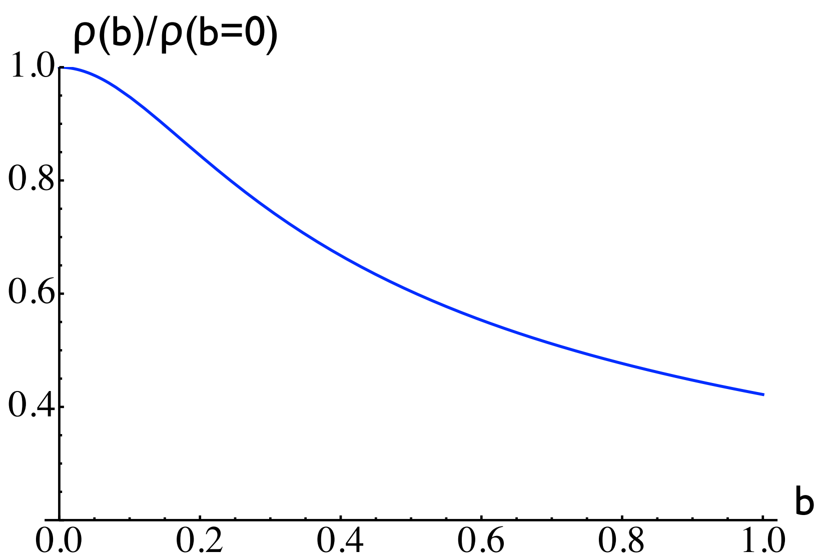
- Non perturbative in warping
- Correction to Dirac physics
- Possible to probe experimentally $$b = \frac{\lambda E_F^2}{2(\hbar v_F)^3}$$

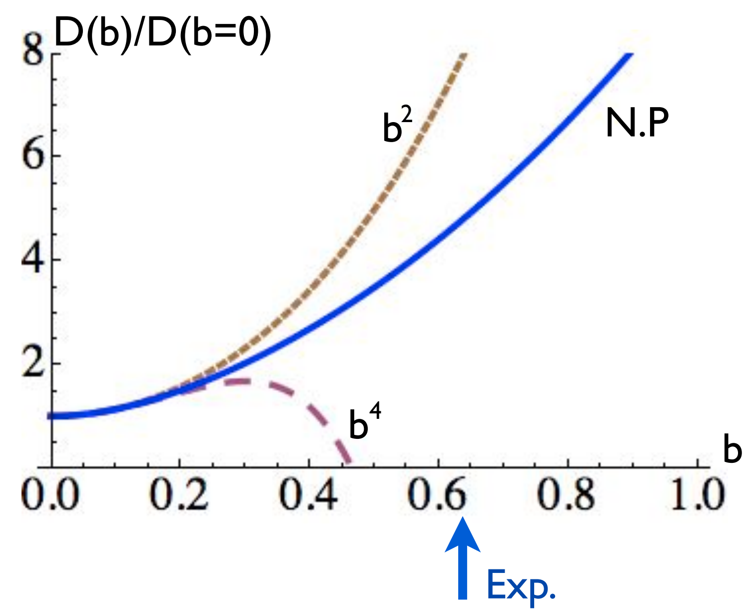




Non perturbative results

- Einstein relation $\sigma = e^2 \rho D$
- Opposite effects
- Strong effect on diffusion constant w.r.t. Dirac physics





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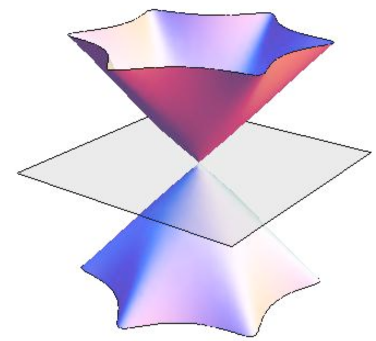

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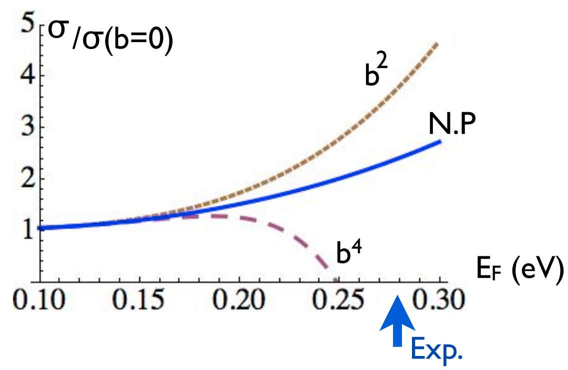
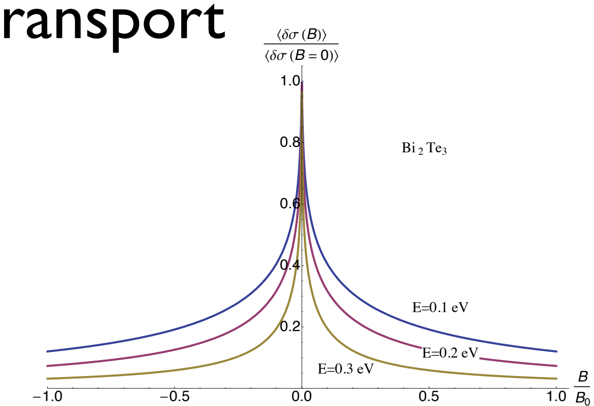


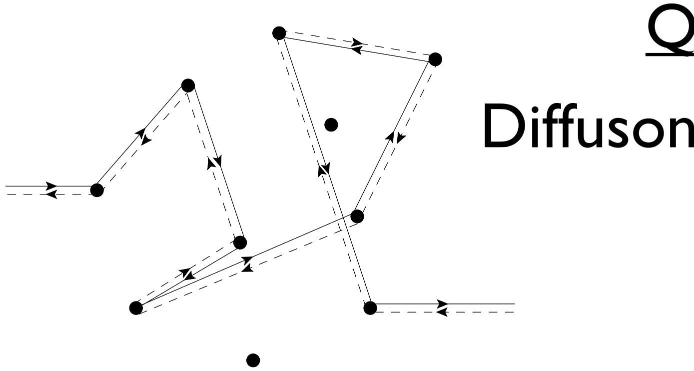
- Coherent transport

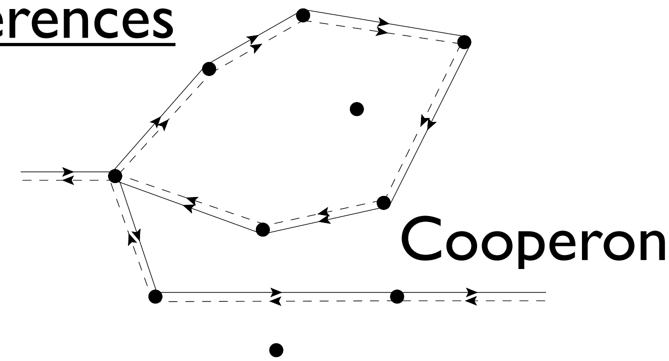


Coherent transport

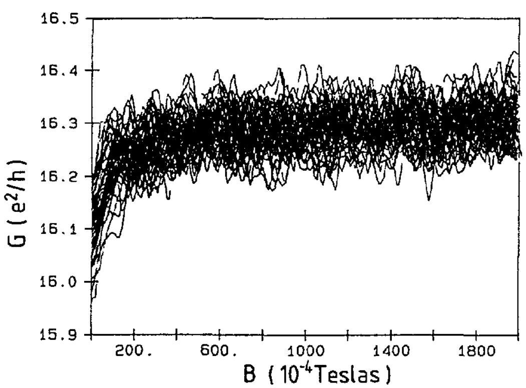
- Phonons : finite coherence time τ_φ
  Mesoscopic physics : low T (τ_φ ↗), small samples

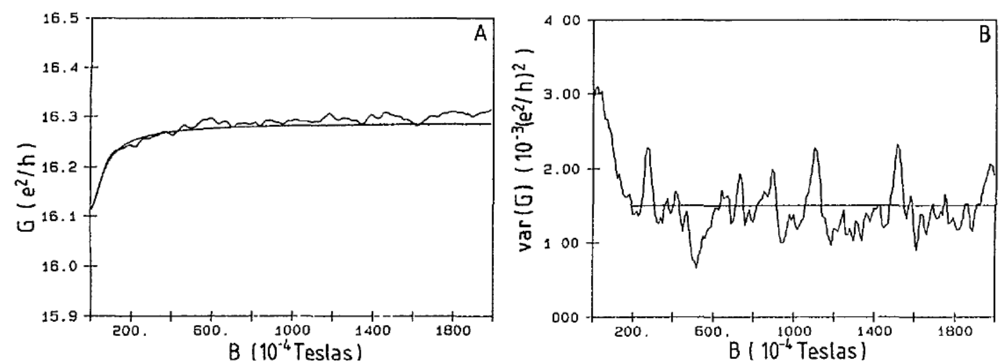
<u>Quantum interferences</u>





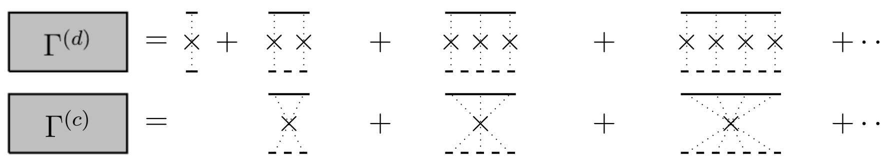
- Universal values : weak (anti)localization / UCF



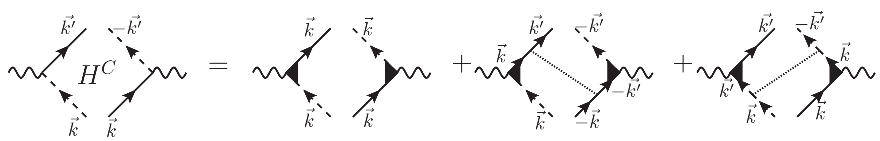


Coherent transport : diagrammatics

- Interferences effects : 2 diffusive modes

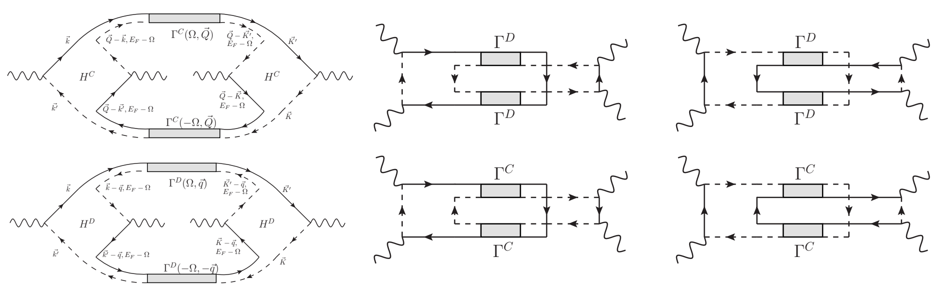


- Weak anti-localization



$$\langle \delta\sigma \rangle = \frac{e^2}{\pi\hbar} \int_{\vec{Q}} \frac{1}{Q^2}$$

- Conductance fluctuations



$$\langle \delta\sigma^2 \rangle = 12 \left(\frac{e^2}{h} \right)^2 \frac{1}{V} \int_{\vec{q}} \frac{1}{q^4}$$

Same results as non-relativistic electrons
with random spin-orbit coupling !

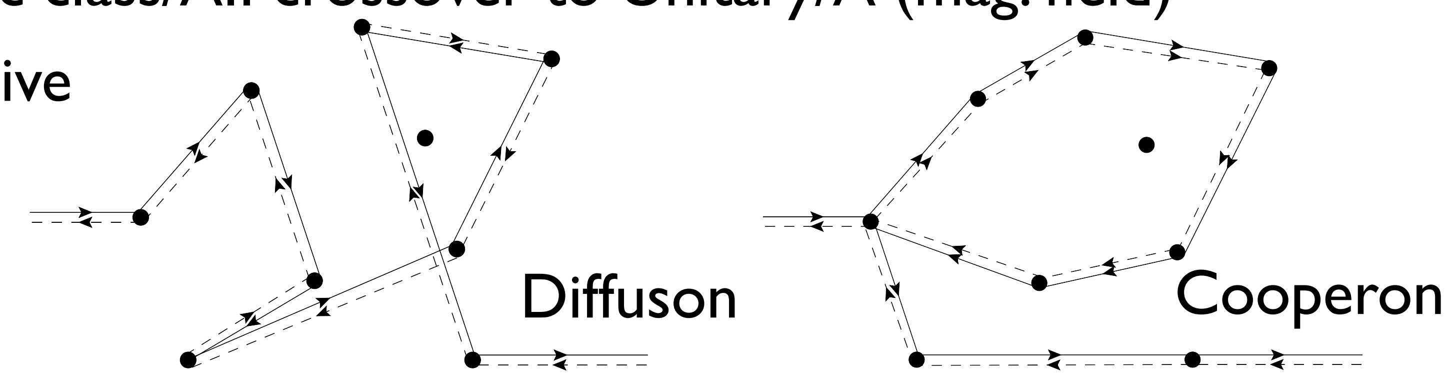
Anderson problem

- Coherent metal + weak disorder : Anderson problem
  - Universality classes for transition (strong disorder) : universal metallic properties (weak disorder)
  - Time Reversal Symmetry , $\mathcal{H} = \hbar v_F(\vec{\sigma} \times \vec{k}).\hat{z} + \frac{\lambda}{2}(k_+^3 + k_-^3)\sigma^z + V(\vec{r})$
  - $T^2 = -1$

	Symmetry			$d-1$			
	T	P	C	0	1	2	
Wigner - Dyson Classes							
A	0	0	0	0	$\mathbb{Z}$	0	Unitary
AI	1	0	0	0	0	0	Orthogonal
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	Symplectic

- Symplectic class/AII crossover to Unitary/A (mag. field)

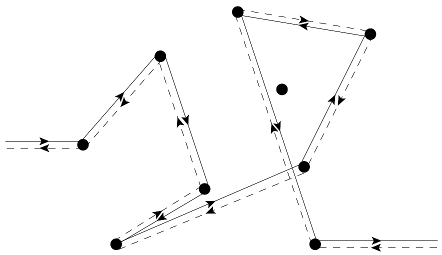
2 / 1 diffusive modes



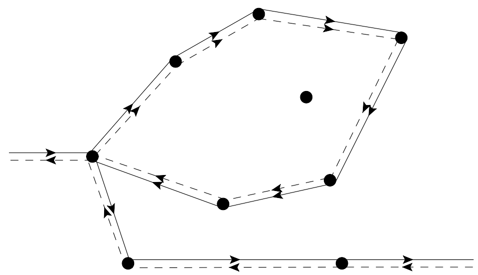
Symplectic and unitary classes results

Symplectic class, TRS — Unitary class

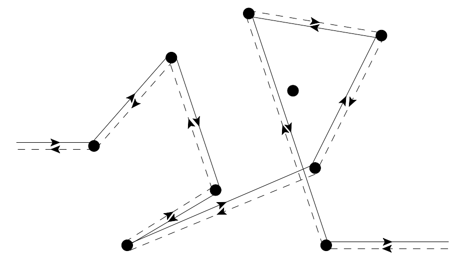
- Diffusive modes



- Diffuson



- Cooperon



- Diffuson

- Weak Anti Localization (WAL)

Symplectic class, TRS:

$$\langle \delta\sigma \rangle = \frac{e^2}{\pi\hbar} \int_{\vec{Q}} \frac{1}{Q^2}$$

Unitary class:

$$\langle \delta\sigma \rangle = 0$$

- Conductance fluctuations

Symplectic class, TRS:

$$\langle \delta\sigma^2 \rangle = 12 \left(\frac{e^2}{h}\right)^2 \frac{1}{V} \int_{\vec{q}} \frac{1}{q^4}$$

Unitary class:

$$\langle \delta\sigma^2 \rangle = 6 \left(\frac{e^2}{h}\right)^2 \frac{1}{V} \int_{\vec{q}} \frac{1}{q^4}$$

Universal results : specificity of Dirac in the crossovers

WAL crossover

- Phase coherence length : $L_\phi = \sqrt{D(b)\tau_\phi}$
- Result for $L_\phi \ll L$:

$$\langle \delta\sigma \rangle = \frac{e^2}{\pi\hbar} \ln(\frac{L_\phi}{\ell_e})$$

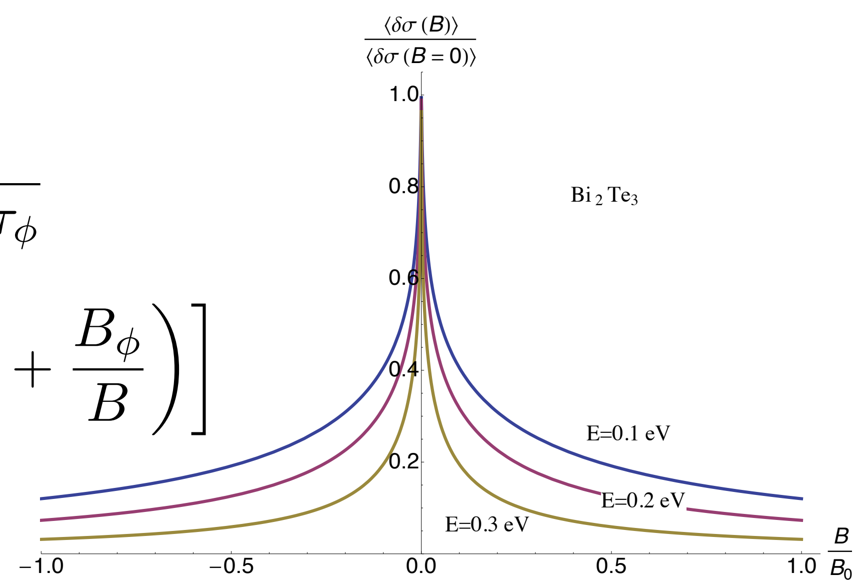
Function of b (or E_F)

- Crossover :

$$B_e(b) = \frac{\hbar}{4eD(b)\tau_e} \qquad B_\phi(b) = \frac{\hbar}{4eD(b)\tau_\phi}$$

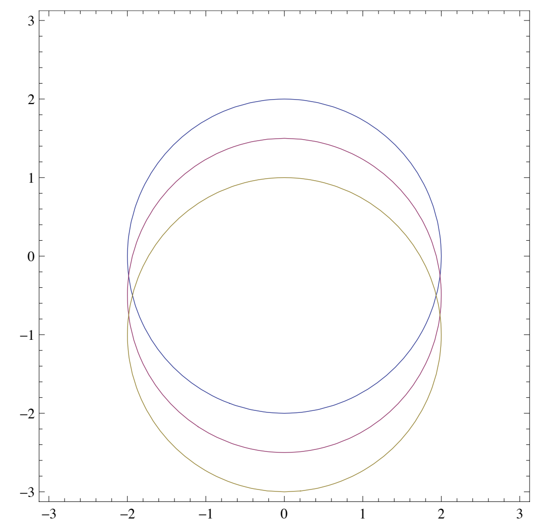
$$\langle \delta\sigma(B) \rangle = \frac{e^2}{4\pi^2\hbar} \left[\Psi\left(\frac{1}{2} + \frac{B_e}{B}\right) - \Psi\left(\frac{1}{2} + \frac{B_\phi}{B}\right) \right]$$

Function of b (or E_F)



In plane Zeeman magnetic field

- Extra term in the hamiltonian : $g\mu_B \vec{B}.\vec{\sigma}$
- Effect in absence of hexagonal warping



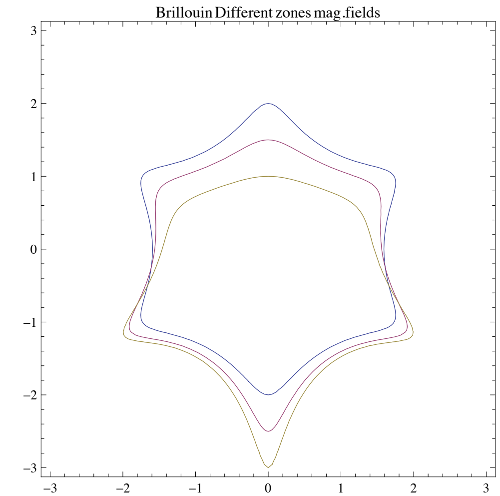
- No change of the scattering probability
- Cooperon stays massless

$$\frac{1}{D(b)Q^2\tau_e - i\omega\tau_e/\hbar}$$

No change in conductivity

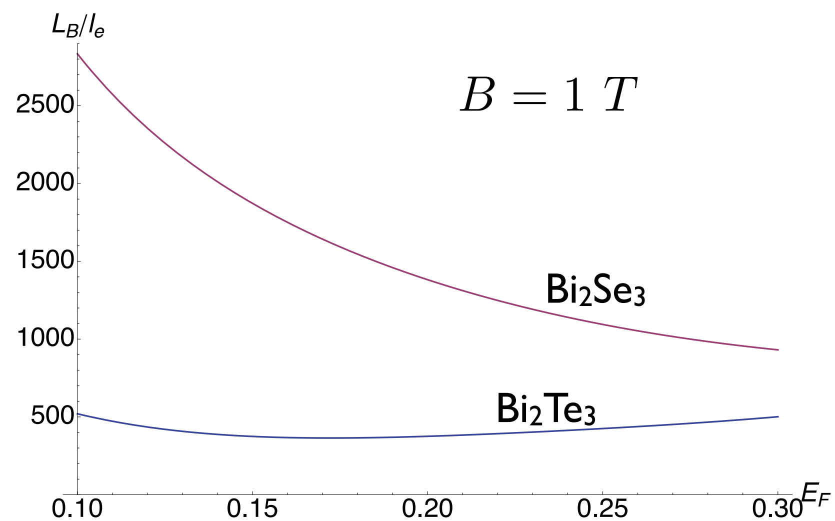
In plane Zeeman magnetic field

- Effect in absence/presence of hexagonal warping



- TRS broken, crossover to unitary class. Cooperon is no longer massless

$$\frac{1}{D(b)Q^2\tau_e - i\omega\tau_e/\hbar + m(b,B)}$$

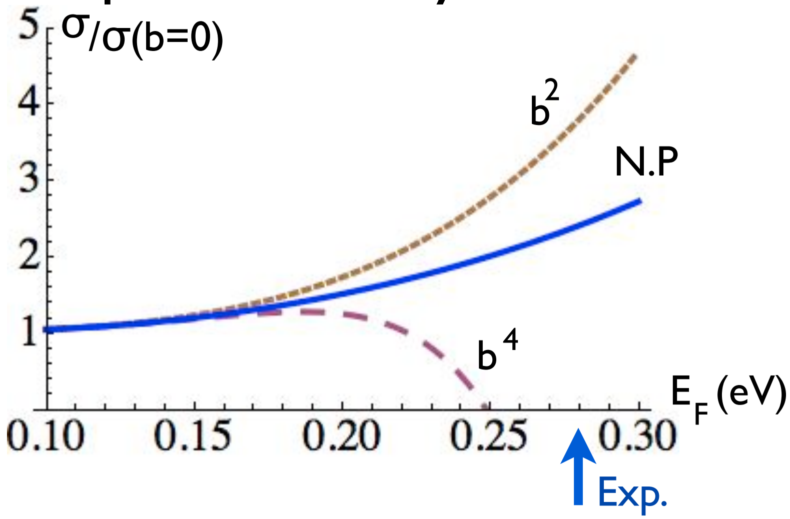


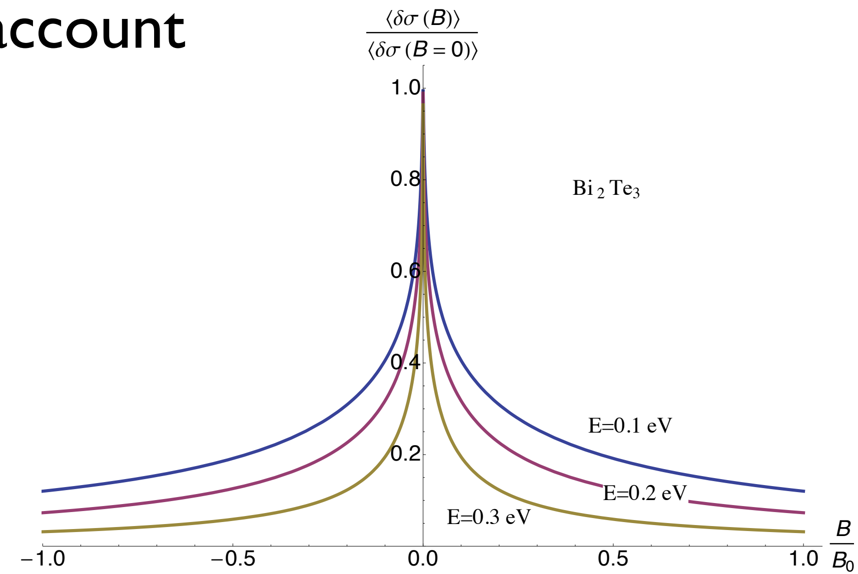
Magnetic length

$$L_B = \sqrt{D(b)\tau_e/m(b,B)}$$

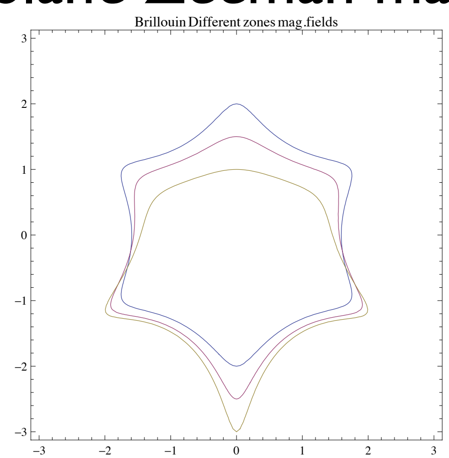
Conclusions

- Hexagonal warping taken into account non-perturbatively





- In-plane Zeeman magnetic field effect



http://arxiv.org/abs/1205.5209